Research Notes for Chapter 14*

The evolution of job shop scheduling is mostly restricted to deterministic models and until now it went through several developmental stages. For completeness, we first summarize our discussion about job shops in our Research Notes for Chapter 10. The job shop problem was first posed by Akers and Friedman (1955) and Akers (1956) provided a graphical solution for the two-job *m*-machine job shop. Conway, Maxwell and Miller (1967) showed that the polynomial solution remains valid even when recirculation is allowed. Jackson (1956) showed that a two-machine job shop without recirculation can be solved with a polynomial algorithm. Garey, Johnson and Sethi (1976) showed that job shops with recirculation are NP-hard even for two machines, whereas a job shop with $m = 3$ is already NP-hard even without recirculation.

Following these early polynomial special-case solutions, important early contributions were made by Giffler and Thompson (1960) and by Roy and Sussman (1962). The former defined active schedules and thus described an important dominant set. Algorithm 14.1 is due to them. The latter presented the now ubiquitous network model with conjunctive and disjunctive arcs.† Indeed, early branch and bound algorithms were based on both these ideas. Branches can be defined by the direction of disjunctive arcs or by Algorithm 14.1.‡ Another early contribution was by Fischer and Thompson (1963), published as a chapter in Muth and Thompson (1963). Among other things, they provided test problems that were later used extensively to test new algorithms. One of those problems, known as the 10 × 10 (and often attributed to Muth and Thompson, apparently by error), involved ten jobs and ten machines (and thus a total of one hundred operations), resisted all attempts to solve it for over a quarter of a century. Mellor (1966) provides a somewhat provocative qualitative review of the state of the art in job shop scheduling at the end of the first decade (and citations for earlier reviews). Some of the practical reservations he makes are, arguably, still valid today.

The 10 × 10 problem was not solved until the development of the shifting bottleneck algorithm by Adams, Balas and Zawack (1988). Indeed, we can count that development, alongside concurrent and subsequent similar ones by Carlier and Pinson (1989) and (1994) and by Dauzere-Peres, and Lasserre (1993), as the next important stage. As we mention in the chapter, all shifting bottleneck algorithms for job shops rely on he HBT solution of Carlier (1982). We also remind readers the connection to Potts

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† We defined disjunctive arcs in the chapter as arcs whose orientation is not fixed in advance but is rather determined by our sequencing decisions. Conjunctive arcs are those whose orientations *are* prefixed by the technological precedence relationships.

‡ In this connection, it is interesting to note that the branching scheme of the shifting bottleneck algorithm is by disjunctive arcs but the structure of the algorithm ensures that only active schedules are ever tested.

(1980) that we discussed in our Research Notes for Chapter 10. Although additional improvements are still being developed, the shifting bottleneck algorithm provided a very significant breakthrough. The number of operations for which we can now solve has tripled relative to the earliest approaches. However, we may have reached a new level of resistance. At least for now, only heuristics can solve large problems. This observation brings us to the next stage.

As we indicated in the chapter, the shifting bottleneck algorithm was used right from the start both as an optimizing procedure and as a heuristic approach. Heuristics were used early on for the job shop problem, not only for the static version but also for the more practical dynamic version (e.g., see Gere, 1966; Panwalkar and Iskander, 1977). As noted by Mellor, that was a matter of necessity. Much of our knowledge about the job shop is indeed based on simulation studies that test simple heuristics, typically based on priority rules. We discuss this topic in more depth in Chapter 15. Following the introduction of the shifting bottleneck algorithm, modern heuristics were applied to the basic job shop problem with renewed energy. These approaches fell into two main groups: heuristics based on the shifting bottleneck algorithm itself, which for a while dominated the field, and heuristics based on neighborhood searches (among which we also count simulated annealing, tabu search and genetic algorithms). As stated in the chapter, these search approaches define the present state of the art in heuristic job shop scheduling. The picture is similar in deterministic project scheduling (see Chapter 17). One may ask why it took so long for these effective heuristics to emerge—for instance, no early heuristic gave the optimal solution of the 10×10 problem. Contrast that with the case of the single-machine weighted tardiness problem that we discussed in our Research Notes for Chapter 4. There, a Dynasearch application was capable of providing solutions that later proved to be optimal for all open library problems published up to that time. The answer is that early search heuristics did not avoid searching infeasible sequences. The key to the improved performance was the development of the modified neighborhoods of the type we discuss in the chapter and in our Reseach Notes for Chapter 4. Aarts, van Laarhoven, Lenstra and Ulder (1994) and Vaessens, Aarts and Lenstra (1996) discuss and compare neighborhood search approaches for the job shop. Dell'Amico and Trubian (1993) present a tabu search model. They are also the source of the results we reported about the restricted neighborhood (page 345), namely that only interchanges that belong to it can reduce the makespan in one step and that it does not guarantee connectivity. Nowicki and Smutnicki (2005) improve on a tabu search algorithm that they first presented in Nowicki and Smutnicki (1996). They use the restricted neighborhood and compensate for the lack of connectivity by using multiple seeds. In the chapter, we treated that algorithm as the most effective approach currently available. We also cautioned that the effectiveness of this algorithm cannot be fully attributed to tabu search as such, because the actual algorithm borrows heavily from other approaches and because it relies on a very clever bound computation that could be adopted under any other search technique. For instance, a formerly highly competitive application relied on simulated annealing (Van Laarhoven, Aarts and Lenstra, 1992). We already discussed this issue in our Research Notes of Chapter 4.

Whereas not too much research was done on stochastic models for job shops, robust scheduling models designed to achieve robust sequences, which are relatively stable in the face of random disturbances, have received some attention. We discussed robust scheduling in general in our Research Notes of Chapter 7. Mehta and Uzsoy (1998) rely on that approach for job shops. As for stochastic models, results tend to rely on generalizing insights from the project scheduling arena. As such, they employ heuristics based on no-wait API-based dispatching; that is, the next operation is selected among those available such that no other schedulable operation would be more attractive right ahead of it. In addition, these models rely both on the independence assumption and on the normal distribution. Nonetheless, they also address safe scheduling objectives similar to those known from the project scheduling field. One such model, with service level constraints, was proposed by Golenko-Ginzburg, Kesler and Landsman (1995). A similar approach was presented by Golenko-Ginzburg and Gonik (2002). Those problems rely on given due dates and therefore they may not have a feasible solution. For instance, accept-reject decisions are not involved, so all jobs must be processed and yet all jobs must meet prescribed service levels. A more recent model—Laslo, Golenko-Ginzburg and Keren (2007)—addresses a virtual job shop composed of rented resources with responsibility to perform a given set of jobs. This model adopts the economic approach. Job tardiness costs include both a fixed element incurred once tardiness takes place as well as a proportional element. Earliness cost is modeled by a rental charge on resources. Because it is based on minimizing economic costs, the model is guaranteed to have a feasible solution. This model fits our framework well. We present similar models in the context of project scheduling in Chapter 18. Finally, simulation studies of the type we cover in the next chapter are inherently useful for stochastic scheduling decisions, with or without safety time. Additional research is needed to address more realistic or more general processing time distributions, such as using the lognormal distribution with linear association. An important research question is to determine whether it is more efficient to utilize API-based dispatching or to add safety time to a deterministic counterpart solution. Dispatching can account for randomness explicitly but considers a limited number of options, whereas using a deterministic counterpart considers more options but sequencing decisions are made without accounting for stochastic variation. However, if we are willing to settle for dispatching in the deterministic case, we should be even more willing to do so in a stochastic environment. (In Chapter 18 we present related models that are based on deterministic-counterpart sequencing decisions.)

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