PREDICTING COMPETITIVE RESPONSE TO A MAJOR POLICY CHANGE:
COMBINING GAME THEORETIC AND EMPIRICAL ANALYSES

by

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This research uses P&G’s value pricing initiative as a context for testing the ability of a game theoretic model to predict competitor and retailer response to a major policy change. We first estimate the response parameters of a demand function for each brand from the period before value pricing was initiated. We then formulate a dynamic Manufacturer-Stackelberg model that includes P&G, a national brand competitor, and a retailer. The model takes P&G’s move as given and prescribes the actions that competitors and the retailer should take with respect to price and promotion. We test the predictive power of the game theoretic model by substituting the estimated response parameters into the model to obtain prescriptions for each competitor and the retailer, and then seeing whether these prescriptions are related to the actual moves taken by competitors and the retailer. We also test the predictive power of two benchmark models. The first is based on the reaction function approach of Leeflang and Wittink (1992) and the second is a simplification of our dynamic model where the retailer is not strategic, i.e., the national brand competitor makes price and promotion decisions assuming that the retailer will apply a fixed price mark-up and promotion pass-through rate.

We find that the game theoretic model, coupled with empirical estimates of its response parameters, has significant predictive power. Covariates such as category advertising, category purchase cycle, and multi-market contact also predict competitive response, but the game-theoretic based prescription is the most important predictor. Further, the model performs better than either benchmark. Overall, the results suggest that this approach is fruitful for predicting competitor responses to major policy changes.
1. INTRODUCTION

The study of competitive response has been significantly enhanced by normative economic models. These models draw on game theory to prescribe the actions firms should take in response to moves by other firms. This approach has been applied both to vertical channel relationships (Choi 1991; Kim and Staelin 1999; Lee and Staelin 1997; Raju, Sethuraman, and Dhar 1995) and horizontal “inter-brand” competition (Lal 1990; Raju, Srinivasan, and Lal 1990). These analyses show, for example, that retailers should decrease prices when national brand cross elasticities increase (Kim and Staelin 1999; Raju, Sethuraman, and Dhar 1995) or that national brands should promote to limit competitive encroachment (Lal 1990).

In recent years, researchers have also developed dynamic game theoretic models that prescribe optimal pricing and promotion policies for manufacturers and retailers based on a forward looking analysis in each period (e.g., Kopalle, Rao, and Assunção 1996; Kopalle, Mela, and Marsh 1999). Such models are particularly important for prescribing promotion decisions because promotion has several dynamic effects, including “wear-out” to repetitive promotions (Lattin and Bucklin 1989; Thompson and Noordweir 1992; Kopalle, Mela, and Marsh 1999; Foekens, Leeflang, and Wittink 1999), stockpiling (van Heerde, Leeflang, and Wittink 2000), and retail forward buying (Blattberg and Neslin 1990).

In summary, game theoretic models have been extensively used to prescribe optimal competitive response. A natural next step is to consider whether game theory combined with empirical analysis can actually predict this response a priori. That is, can we empirically estimate market response parameters, substitute them into a game theoretic model, and then predict how firms will respond to competitor moves. This is an important undertaking for two reasons. First, it provides an opportunity to combine the conceptual rigor of game theory with
the descriptive power of empirical analysis, and shed light on whether and under what circumstances this effort pays off. Second, if there indeed is some predictive power to a combined game theoretic/empirical analysis, this work is potentially of direct use to managers trying to anticipate response to their actions.

The purpose of this paper therefore is to develop a dynamic game theoretic model, “calibrate” it with empirically derived estimates of a demand function that incorporates consumer dynamics, use it to predict competitor and retailer response, and test the predictions against actual response. The context we consider is response to the Procter and Gamble “Value Pricing” strategy (Ailawadi, Lehmann, and Neslin 2001). According to media reports, this strategy entailed P&G making major cuts in promotions and providing a lower everyday price to retailers and consumers (Shapiro 1992). The move was well-publicized and counter to industry trends. It therefore provides a strong exogenous stimulus and a valuable opportunity to see whether we can predict the response of competing national brands and retailers to a major policy change by a market leader.

Previous research suggests that predicting competitive response is quite a challenge. First, Leeflang and Wittink (1992) show that competitive response is not always “simple.” A change by a firm in marketing instrument X may evoke a competitive change in marketing instrument Y (see also Kadiyali, Chintagunta, and Vilcassim 2000 and Putsis and Dhar 1998). Second, Leeflang and Wittink (1996) show that firms may “over-react” or “under-react” to competitive moves. These authors estimated demand elasticities and predicted the “optimal” response to various competitor moves, assuming the responding firm wished to preserve its market share. They found that firms often either over or under reacted, with overreactions being more common (see also Brodie, Bonfrer, and Cutler 1996). Third, it is unclear whether managers actually employ the strategic thinking suggested by game-theoretic models. Montgomery, Moore, and Urbany (2003) found that managers often do not consider competitor reactions when deciding on their own move. Interestingly, however, they are more likely to do
so for major, visible decisions, especially pricing related ones. So, the predictive ability of a
game theoretic model is definitely worth testing but it is by no means a given.

The specific game theoretic model we consider is the Manufacturer-Retailer Stackelberg
game. Since it was introduced to the marketing literature by McGuire and Staelin (1983), this
model has been used by several researchers (e.g., Choi 1991; Jeuland and Shugan 1988; Kopalle,
Mela, and Marsh 1999; Kim and Staelin 1999). The game structure reflects the natural temporal
flow of the product in that the manufacturer’s decisions precede that of the retailer. Empirical
support for this type of channel interaction is not unequivocal. Kadiyali, Chintagunta, and
Vilcassim (2000) find that pure forms of traditional games are not supported in their empirical
analysis. On the other hand, based on interviews with a retailer and a manufacturer, Kopalle et
al. (1999) suggest that a Stackelberg game in which the manufacturer moves first is a good
approximation of the manufacturer-retailer interaction. And, more recently, Sudhir (2001)
provides empirical evidence in support of this type of game.

Our model places the Stackelberg game in a dynamic channel interaction framework
similar to Kopalle, Mela, and Marsh (1999), but it is significantly more comprehensive in that it
endogenizes (a) the national brand competitor’s price and promotion decisions; (b) the retailer’s
price decision for P&G, the national brand competitor, and the private label; (c) the retailer’s
private label promotion decision; and (d) the retailer’s forward buying decision for P&G and the
national brand competitor.

The result is a “dynamic structural model” – dynamic in the sense of including temporal
response phenomena and decision-making over time; structural in the sense of modeling the
process by which decisions are made by profit-maximizing agents in a Stackelberg framework.
Such structural models are particularly appropriate in the case of a major policy change because
they are better suited to predict how agents adapt their decisions to a new “regime,” compared to
reduced-form models that attempt to extrapolate from the past (see Keane 1997). The cost,
however, is added complexity plus the difficult decision of how all-encompassing to make the
model. To shed light on these issues, we compare the predictive ability of our model with two benchmarks. The first is the reaction function approach (Leeflang and Wittink 1996). Reaction functions are reduced-form models and supposedly are better suited to short-term incremental changes than to major policy changes (Abeele 1994), but this is an empirical question to be investigated. The second is a dynamic model with a simplification in that the retailer is considered to be non-strategic, applying a constant mark-up. Considering this model gets at the question of how detailed the structural model should be.

We conduct our analysis for a local market because packaged goods companies are known to adapt trade deal policies to local conditions and both competitor and retail reaction can vary substantially across markets (e.g., Blattberg and Neslin 1990; Kadiyali, Chintagunta, and Vilcassim 2000). Doing an aggregate analysis at the national level would obfuscate these differences. We use the scanner database of the Dominicks grocery chain in Chicago and wholesale price and trade deal data for the Chicago market from Leemis Marketing Inc.

The results show that indeed, a game theoretic model combined with strong empirics has predictive power. Our model’s prescription of how competitors should change wholesale price and trade dealing in response to P&G’s move is a significant predictor of the actual changes made by competitors. Similarly, our model’s prescription of how the retailer should change retail prices of all brands and retail dealing of the private label is a significant predictor of the actual changes made by Dominicks in the Chicago market. Competitor characteristics such as multi-market contact, and category characteristics such as average purchase cycle also help to predict actual changes, over and above our economic model prescriptions. The results also show that our model has better predictive ability than either of the benchmark models. This suggests that, in the context of a major pricing policy change, managers’ actions are more consistent with strategic competitive reasoning than with an extrapolation of past reactions into the future, or with ignoring the anticipated reaction of the retailer.
The paper proceeds as follows. We describe the game theoretic model and our dynamic optimization approach in section 2 and the two benchmark models in section 3. Section 4 describes the data used in our empirical analysis and section 5 presents our results. We conclude with a discussion and implications for researchers and managers in section 6.

2. GAME THEORETIC MODEL

2.1 Demand Function

Unit store sales of Brand $i$ in week $t$, $S_{it}$, are given by:

$$S_{it} = \alpha_i + \sum_{k=1}^{K} \beta_{ik} R_{Pt} + \sum_{k=1}^{K} \gamma_{ik} R_{Dt} + \delta_{1i} C_{Dit} + \delta_{2i} (C_{Dit})(R_{Dt})$$  \hspace{1cm} (1)

$$C_{Dit} = \lambda_i C_{Dit-1} + (1-\lambda_i)R_{Dt-1}$$  \hspace{1cm} (2)

where $i, k = 1, 2, 3, .. K$, and $K$ is the number of brands in the category;

$R_{Pt} =$ Regular retail price per unit of Brand $k$ in week $t$;

$R_{Dt} =$ Retail deal amount for Brand $k$ in week $t$;

$C_{Dit} =$ Cumulative dealing, i.e., exponentially smoothed average of past retail deal amounts, for Brand $i$ in week $t$;

$\lambda_i =$ Exponential smoothing parameter for the cumulative dealing of Brand $i$;

$\alpha_i =$ Intercept for brand $i$;

$\beta_{ik} =$ Effect of Brand $k$’s retail price on Brand $i$’s sales;

$\gamma_{ik} =$ Effect of Brand $k$’s retail deal amount on Brand $i$’s sales;

$\delta_{1i} =$ Effect of cumulative dealing of Brand $i$ on sales of Brand $i$;

$\delta_{2i} =$ Effect of cumulative dealing of Brand $i$ on the effectiveness of its current retail deal.

Note we distinguish between consumer response to regular price changes and temporary deal price cuts (Guadagni and Little 1983; Mulhern and Leone 1991), and allow for separate cross-price and cross-deal effects of each competitor. We also capture stockpiling (van Heerde,
Leeflang, and Wittink 2000) and promotion wear-out, i.e., a possible reduction in the impact of a current deal if there has been a stream of deals in the recent past (Lattin and Bucklin 1989; Thompson and Noordeweir 1992; Foekens, Leeflang, and Wittink 1999). The $CD$ term reflects stockpiling and the interaction term, $(CD)(RD)$, reflects wear-out (see Kopalle et al. 1999).

2.2 Channel Structure and Decision Variables

We model the channel structure as a dynamic Manufacturer-Retailer Stackelberg game. Some important features of the model are: First, we have a series of Manufacturer-Stackelberg games solved by forward-looking players. We consider two manufacturers, P&G (Brand 1) and a competitive national brand (Brand 2), selling through a common retailer. The retailer also sells a private label brand (Brand 3). Second, manufacturer and retailer decisions are intertwined though the manufacturer is the Stackelberg leader in each period. In a given period, the manufacturer takes into consideration how the retailer will react in this period as well as the maximum profit he (the manufacturer) will be able to make in the series of Stackelberg games in subsequent periods, conditional on his decision in this period. In turn, the retailer takes into consideration the manufacturer’s decisions in this period as well as the maximum profit he (the retailer) will be able to make in the subsequent series of Stackelberg games, conditional on his decision in this period. Third, both manufacturer and retailer take into account dynamic responses by consumers in the forms of stockpiling and/or promotion wearout. Fourth, both players develop expectations about future actions that are fulfilled in equilibrium because, in equilibrium, neither player has an incentive to deviate from their chosen strategy.

Since P&G’s move was first-and-foremost a pricing and promotion strategy, we consider competitor reactions in terms of (a) wholesale price and (b) what deal amount, if any, to offer. The retailer decides (a) whether and how much to order from each manufacturer, (b) whether to order enough to satisfy future demand as well as current demand (forward buying), (c) retail
prices of P&G, the national brand competitor, and the private label, and (d) what retail deal amount, if any, to offer on the private label.¹

We consider P&G’s actions to be exogenous, and hence not decision variables. This is defensible because the company clearly announced its intention to implement value pricing (e.g., Shapiro 1992) and then went on to do so. It is possible that P&G subsequently modified its strategy in response to the reaction it encountered in the market. However, incorporating such response by P&G would make our model intractable. Therefore, we conduct our analysis in a period during which such endogenous response by P&G is less likely to have occurred. We discuss this issue in more detail in Section 4.

We follow Silva-Risso, Bucklin, and Morrison (1999) in assuming that when the retailer orders from a manufacturer during a trade deal period, he provides a constant percentage of the trade deal amount to the consumer in that period. However, since we endogenize forward-buying by the retailer, the “effective” pass-through, i.e., the amount that the retailer sells on deal to the consumer relative to the amount he buys on deal from the manufacturer, is endogenized.

2.3 Retailer’s Objective Function and Dynamic Program Formulation

The retailer’s and the manufacturer’s objectives are to maximize their respective sum of profits over a finite time horizon, 1 to \( T \). Given a manufacturer’s wholesale price and trade deal amount during \( t = 1, \ldots, T \), the retailer chooses prices, \( RP_{1t}, RP_{2t}, RP_{3s}, \) whether to place an order with the manufacturer during each period; if so, what should be the order quantities, \( X_{1t} \) and \( X_{2t} \), and retail deal amount, if any, for the private label, \( RD_{3t} \). Therefore, the retailer’s objective function is:

\[
\max_{RP_{1t},RP_{2t},RP_{3s},RD_{3t},X_{1t},X_{2t}} \sum_{t=1}^{T} \left[ \sum_{i=1}^{2} \left( (RP_{i} - kD_{i} I_{i}) S_{it} - (WP_{it} - D_{it}) X_{it} - h_{i} INV_{it} \right) + (RP_{3t} - VC - RD_{3t}) S_{3t} \right] \tag{3}
\]

where:

¹ We assume that the retailer does not engage in diversion of product.
² We don’t discount future profits because we consider a fairly short time horizon of 15 weeks, but that can be easily incorporated through a discount factor in Equation (3). Also, we assume without loss of generality that fixed costs are zero. This is equivalent to saying that contribution to fixed costs is being maximized.
Inventory of Brand $i$ in week $t$, $INV_{it} = INV_{i,t-1} + X_{i,t-1} - S_{i,t-1}, \ i = 1, 2$

$WP_{it} = $ Wholesale price of Brand $i$ in week $t$

$D_{it} = $ Trade deal offered by manufacturer $i$ in week $t$

$k = $ Retailer pass-through of manufacturer trade deal

$h_i = $ Retailer’s per unit, per period inventory holding cost for Brand $i$

$VC = $ Variable cost of the private label to the retailer

$I_{it} = $ Order indicator variable for brand $i$ in period $t$, i.e., $= 1$ if $X_{it}>0$; $0$ if $X_{it}=0$.

Sales, $S_{it}$, for each brand are given by the demand function in equation (1). In equation (3), the first three terms apply to the national brands ($i=1,2$). The first term, $(RP_{it} - kD_{it}I_{it})S_{it}$ represents revenues the retailer generates for Brand $i$ in period $t$, where $kD_{it}$ is the deal amount that the retailer provides to consumers in a week when (s)he orders ($I_{it}=1$) under a trade deal of $D_{it}$ from the manufacturer. The second term, $(WP_{it}-D_{it})X_{it}$, represents costs incurred by the retailer for Brand $i$ in period $t$. The third term, $h_iINV_{it}$, represents the retailer’s inventory cost in period $t$ for Brand $i$. The fourth term, $(RP_{3t} - V - RD_{3t})S_{3t}$ is the retailer’s profit in period $t$ from the private label (Brand 3). We assume initial inventory levels are zero and the salvage value of any inventory held at the end of the time horizon is zero.\(^3\)

**Dynamic Programming Formulation**

For a given manufacturer’s wholesale price and trade deal amount in period $t$, we formulate the optimization as a dynamic programming problem, where the state space consists of $L_{it}$ (the period before $t$ in which an order was last placed for Brand $i$), $i = 1, 2$; $WP_{it}$, the wholesale price for Brand $i$ when it was last ordered by the retailer prior to period $t$; $D_{it}$, the trade deal for Brand $i$ when it was last ordered by the retailer prior to period $t$; and $CD_{1t}$, $CD_{2t}$, and $CD_{3t}$, the cumulative retailer deals.

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\(^3\) To allow for non-zero inventory value at the end of the planning horizon, and also to alleviate any end game effects, we run the optimization for $T+x$ periods and consider only the decisions in the planning horizon, $T$. 

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For such problems, Hall, Kopalle, and Krishna (2002) and Wagner and Whitin (1958) have derived “zero inventory ordering” and “period-covering” properties that show the retailer places orders for a brand only when existing inventory is exhausted, and that orders will cover total demand over some integer number of sequential future periods. In other words, orders do not cover a fraction of a period’s demand – they either cover none or all of the demand for a given period. The number of future periods for which orders are placed in the current period measures the retailer’s forward buying. This result enables us to restate the retailer’s problem in Equation (3) as a dynamic programming problem in discrete time (Stokey and Lucas 1989):

\[
\pi^*_R(L_{it}, WP_{L_{it}}, D_{L_{it}}, CD_{it}, CD_{3t} | WP_{it}, D_{it}, i = 1,2) = \max_{I_{it}, I_{2t}, R_{P_{it}}, R_{P_{2t}}, R_{D_{it}}, R_{D_{2t}}}
\]

\[
\begin{align*}
& I_{2t}(1 - I_{it})[(RP_{1t} - WP_{L_{1t}} + D_{L_{1t}} - h_1(t - L_{it}))S_{1t} + (RP_{2t} - WP_{L_{2t}} + (1-k)D_{2t})S_{2t} \\
& + (RP_{3t} - VC - RD_{3t})S_{3t}] \\
& + I_{2t}I_{it}[(RP_{1t} - WP_{L_{1t}} + (1-k)D_{1t})S_{it} + (RP_{2t} - WP_{L_{2t}} + (1-k)D_{2t})S_{2t} \\
& + (RP_{3t} - VC - RD_{3t})S_{3t}] \\
& + (1 - I_{2t})(1 - I_{it})[(RP_{1t} - WP_{L_{1t}} + D_{L_{1t}} - h_1(t - L_{it}))S_{it} + (RP_{2t} - WP_{L_{2t}} + D_{L_{2t}} - h_2(t - L_{2t}))S_{2t} \\
& + (RP_{3t} - VC - RD_{3t})S_{3t}] \\
& + (1 - I_{1t})(1 - I_{2t})[(RP_{1t} - WP_{L_{1t}} + D_{L_{1t}} - h_1(t - L_{it}))S_{it} + (RP_{2t} - WP_{L_{2t}} + D_{L_{2t}} - h_2(t - L_{2t}))S_{2t} \\
& + (RP_{3t} - VC - RD_{3t})S_{3t}] \\
& + V_{R_{it+1}}(L_{it+1}, WP_{L_{it+1}}, D_{L_{it+1}}, CD_{it+1}, CD_{3t+1}, i = 1,2)
\end{align*}
\]

(4)

where:

\[
\pi^*_R = \text{Retailer’s maximum profit from period } t \text{ through } T \text{ given Brand } i \text{'s wholesale price } (WP_{it}) \text{ and trade deal amount } (D_{it}), \text{ } i=1,2, \text{ in period } t;
\]

\[
I_{it} = \begin{cases} 1 \text{ if the retailer places an order for Brand } i \text{ in period } t \\ 0 \text{ if the retailer carries forward inventory of Brand } i \text{ from period } L \text{ to period } t; \end{cases}
\]

The first four major terms in equation (4) compute current period profit depending on four ordering possibilities in period t (ordering Brand 2 but not Brand 1, ordering both brands, ordering Brand 1 but not Brand 2, and not ordering either brand). If the brand is ordered in
period $t$, the retailer offers to consumers $k\%$ of any trade deal in that period, i.e., when $I_t=1$, $RD_t = kD_t$, and so adds $(1-k)D_t$ to his profit margin. Note that there is no issue of pass-through with the retailer’s private label, Brand 3. If brand $i$, $i=1,2$, is not ordered in period $t$, the retailer supplies demand through product that was forward bought earlier, in which case $D_{t+1}$ is added to its margin if in fact there was a deal offered the last time the brand was ordered. Also if the brand is not ordered in period $t$, the retailer subtracts a holding cost for the inventory it has forward bought to satisfy this period’s demand. The last term, $V_{R_t+1}$, is the maximum profit the retailer can expect to make, given the decisions he makes in period $t$. This is evaluated at the manufacturer’s optimal decisions in period $t+1$ ($WP^*_{2t+1}$ and $D^*_{2t+1}$), given P&G’s actions in $t+1$, $WP_{t+1}$ and $D_{t+1}$, and the retailer’s actions in period $t$. Thus:

$$V_{R_{t+1}}(L_{t+1}, WP_{t+1}, D_{t+1}, CD_{t+1}, CD_{3t+1}, i = 1,2) = \pi^*_{R_{t+1}}(L_{t+1}, WP_{t+1}, D_{t+1}, CD_{t+1}, CD_{3t+1}, i = 1,2 | WP_{t+1}, D_{t+1}, WP^*_{2t+1}, D^*_{2t+1})$$

(5)

The state variables $CD_{jt+1}$ ($j=1,2,3$) are as given in Equation (2). The other state variables in period $t+1$ are given by ($i=1,2$):

$$L_{it+1} = I_{it} + (1-I_{it})L_{it}; \quad WP_{t+1} = I_{it}WP_{it} + (1-I_{it})WP_{t+1}; \quad D_{t+1} = I_{it}D_{it} + (1-I_{it})D_{t+1}$$

(6)

We make three points about the state variables. First, keeping track of $L_{it}$, time of last order of Brand $i$ removes the need to track all possible levels of inventory for Brand $i$, a computationally burdensome state space, while allowing us to uniquely determine the cost to the retailer for that brand in any period. Second, since Brand 2 is strategic, the retailer determines his optimal ordering of Brand 2 for all possible values of that brand’s wholesale price and deal amount decisions, while taking into consideration P&G’s actions, i.e., $WP_{it}$ and $D_{it}$. In contrast, P&G’s actions are exogenous and assumed to be known to Brand 2 and the retailer, so the

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4 Note that the wholesale prices, retail prices, and deal amounts differ depending on which of the four order scenarios the retailer undertakes. Therefore we cannot factor out $S_{it}$, for example, in writing equation (4).
retailer can solve for $CD_{1t}$ and $I_{1t}$ ex ante.\footnote{Since P&G made the nature of its value pricing move clear in public announcements, we are confident that competitors knew the general thrust of the policy (i.e., reduction in deals and a lower everyday price). They may also have known exact week by week actions a quarter ahead through their salespeople and retailer contacts. Our best approximation to what competitors knew is what P&G actually did. But, to the extent that this is inaccurate the accuracy of our predictions will suffer.} Third, $L_{1t}$ and $L_{2t}$ take on values 1 through T-1 representing weeks; $WP_{l_{2t}}$, $D_{l_{2t}}$, $CD_{2t}$, $CD_{3t}$ are discretized at ten levels around their respective initial values to keep computing time manageable; and $WP_{l_{it}}$, $D_{l_{it}}$, $CD_{it}$ are based on P&G’s actual values.

### 2.4 Competitive Manufacturer Objective Function and Dynamic Program Formulation

In each period $t$, the profit for Brand 2 depends on whether the retailer buys now from Brand 2 or carries forward Brand 2 from the last purchase order. Hence Brand 2’s objective function is:

$$
\max_{WP_{l_{2t}}, D_{l_{2t}}} \sum_{t=1}^{T} \{I_{2t}(WP_{l_{2t}} - VC - D_{l_{2t}})S_{2t} + (1 - I_{2t})(WP_{l_{2t}} - VC - D_{l_{2t}})S_{2t}\} 
$$

(7)

The first term, $I_{2t}(WP_{l_{2t}} - VC - D_{l_{2t}})S_{2t}$, is Brand 2’s profit if the retailer places an order in period $t$ at the given wholesale price and trade deal amount. The second term, $(1 - I_{2t})(WP_{l_{2t}} - VC - D_{l_{2t}})S_{2t}$, is Brand 2’s profit if the retailer fulfills demand for Brand 2 in period $t$ from inventory carried forward from a past period, $L_{2t}$.\footnote{Note that as in equation (4), the wholesale and deal amounts in period $t$ will differ depending on whether the retailer orders. Therefore, it is not appropriate to factor out $S_{2t}$ in equation (7).} Brand 2 infers retailer actions by considering the retailer’s objective of maximizing his sum of category profits over the time horizon, and then sets his wholesale price and trade deal amount to maximize his own profit.

Following the “zero inventory ordering” and “period-covering” properties discussed in Section 2.3, we can reformulate the manufacturer’s problem in Equation (7) also as a dynamic programming problem in discrete time with state vector \{\(L_{it}, WP_{l_{it}}, D_{l_{it}}, CD_{it}, CD_{3t}, i=1,2\)\}:
where $V_{Mt}$ is the manufacturer’s maximum profit from period $t$ through $T$ and the other variables are as defined earlier.

2.5 Solving for Optimal Decisions

We now provide an overview of how we solve these optimization problems jointly. Please see the Appendix for more details. In each period, we have a Stackelberg game with the manufacturer as the leader and the retailer as the follower, but to solve that game for each period, we need to take into account future profits that depend on the actions taken in each period. The optimization entails the following steps:

1. Starting in period $T$, we optimize the retailer’s profit for all possible values of his state variables and possible manufacturer decisions.

2. Still for period $T$, we optimize the manufacturer’s profit for all possible values of his state variables, taking into account what Step 1 tells us the retailer will do in period $T$ depending on the manufacturer’s decisions.

3. For period $T-1$, we optimize the retailer’s profit in that period (for all possible values of his state variables and possible manufacturer decisions), plus the future profits the retailer expects to make depending on the decisions he makes in period $T-1$. We know those future profits because we have solved the period $T$ case in Steps 1 and 2 above.

4. Still for period $T-1$, we optimize the manufacturer’s profits, taking into account how the retailer will respond in period $T-1$ depending on the manufacturer’s decisions, plus the profits the manufacturer can expect to make in the future depending on the decisions he makes in period $T-1$. We know how the retailer will respond in period $T-1$ from Step 3, and we know the manufacturer’s future expected profits because of Steps 1 and 2.

\[
V_{Mt}(L_{it}, WP_{Lt}, D_{Lu}, CD_{it}, CD_{3r}, i = 1, 2) =
\max_{W_{2t}, D_{2t}} \left\{ \left( I_{2t}(WP_{2t} - VC - D_{2t})S_{2t} + (1 - I_{2t})(WP_{2t} - VC - D_{2t})S_{2t} \right) \right\}
\]

(8)
In this way, the retailer’s decision problem is embedded within the manufacturer’s decision problem, and *vice versa*. The retailer’s decisions in any period are predicated upon the manufacturer’s wholesale price and trade deal amounts in that period as well as what the retailer expects the manufacturer’s wholesale price and trade deal amounts to be in future periods. The manufacturer’s decisions in any period are predicated on what the retailer will decide in terms of pricing and forward buying in that period depending on what actions the manufacturer takes, plus what the manufacturer expects the retailer’s future pricing and forward buying to be. Thus, each player forms expectations about what the other player will do in future periods based on what is optimal, and these expectations are fulfilled in equilibrium because neither player has an incentive to defect from the optimal strategy.

Note that the number of periods for which the retailer forward buys Brand $i$ is a function of retailer costs for Brand $i$ as well as what the retailer expects the manufacturer to offer in the future with respect to wholesale prices and trade deal amounts. Thus, retailer forward buying, retail pricing decisions for the three brands, retail deal amount for the private label, Brand 2’s wholesale price and trade deal amount are all dynamic, interdependent decision variables.

### 3. BENCHMARK MODELS

#### 3.1 Reaction Functions

We specify four reaction functions, two for the national brand manufacturer and two for the retailer. There is one manufacturer reaction function for wholesale price and one for wholesale deal amount for every national brand. Similarly, there is one retailer reaction function for retail price and another for retail deal amount for P&G, every other national brand, and the retailer’s private label. The specification of the four reaction functions is:

\[
\ln\left(\frac{WP_i}{WP_{i-1}}\right) = a_{1i} + \sum_{k=1}^{K} b_{1ik} WP_{kt} + \sum_{k=1}^{K} c_{1ik} \tilde{D}_{kt} + \sum_{k=1}^{K+1} d_{1ik} \tilde{RP}_{kt} + \sum_{k=1}^{K+1} e_{1ik} \tilde{RD}_{kt} + \varepsilon_{1it} \tag{9}
\]

\[
D_{it} - D_{i(t-1)} = a_{2i} + \sum_{k=1}^{K} b_{2ik} WP_{kt} + \sum_{k=1}^{K} c_{21ik} \tilde{D}_{kt} + \sum_{k=1}^{K+1} d_{2ik} \tilde{RP}_{kt} + \sum_{k=1}^{K+1} e_{2ik} \tilde{RD}_{kt} + \varepsilon_{2it} \tag{10}
\]
\[
\begin{align*}
\ln \left( \frac{R_{it}}{R_{it-1}} \right) &= a_{3i} + \sum_{k=1}^{K} b_{3ik} \tilde{W}_{it} + \sum_{k=1}^{K} c_{3ik} \tilde{D}_{it} + \sum_{k=1}^{K} d_{3ik} \tilde{R}_{it} + \sum_{k=1}^{K} e_{3ik} \tilde{RD}_{it} + \\
&+ \sum_{k=1}^{K} f_{3ik} \ln \left( \frac{W_{it}}{W_{it-1}} \right) + \sum_{k=1}^{K} g_{3ik} (D_{it} - D_{it-1}) + \epsilon_{3it} \\
\begin{align*}
RD_{it} - RD_{it-1} &= a_{4i} + \sum_{k=1}^{K} b_{4ik} \tilde{W}_{it} + \sum_{k=1}^{K} c_{4ik} \tilde{D}_{it} + \sum_{k=1}^{K} d_{4ik} \tilde{R}_{it} + \sum_{k=1}^{K} e_{4ik} \tilde{RD}_{it} + \\
&+ \sum_{k=1}^{K} f_{4ik} \ln \left( \frac{W_{it}}{W_{it-1}} \right) + \sum_{k=1}^{K} g_{4ik} (D_{it} - D_{it-1}) + \epsilon_{4it}
\end{align*}
\end{align*}
\]  

where \((i = 1, 2, \ldots, K)\):

\[
\tilde{W}_{it} = \lambda_{1E} \tilde{W}_{it-1} + (1 - \lambda_{1E}) \ln \left( \frac{W_{it-1}}{W_{it-2}} \right) \\
\tilde{D}_{it} = \lambda_{1E} \tilde{D}_{it-1} + (1 - \lambda_{1E}) (D_{it-1} - D_{it-2}) \\
\tilde{R}_{it} = \lambda_{2E} \tilde{R}_{it-1} + (1 - \lambda_{2E}) \ln \left( \frac{R_{it-1}}{R_{it-2}} \right) \\
\tilde{RD}_{it} = \lambda_{2E} \tilde{RD}_{it-1} + (1 - \lambda_{2E}) (RD_{it-1} - RD_{it-2})
\]

\(E = 1, 2, 3, 4\) for equations 9, 10, 11, and 12 respectively

\(\lambda_{1E}\) = Geometric decay parameter for past wholesale prices and wholesale deal amounts

\(\lambda_{2E}\) = Geometric decay parameter for past retail prices and retail deal amounts.

This is a fairly comprehensive specification in which we allow for cross-instrument reactions because Leeflang and Wittink (1992) find that reactions are not necessarily “simple”. We also allow for the possibility that manufacturers react to the past actions not just of competing manufacturers, but also of the retailer, and the retailer takes account not only of his own past actions but also the past and current actions of the manufacturers.\(^7\) Finally, we follow Leeflang and Wittink (1992; 1996) in taking the first difference of log of price and the first difference of deal amount, and Kopalle et al. (1999) in using a parsimonious decay specification for the impact of past prices and deals instead of multiple lags. This benchmark model, while it allows for fairly complex cross-instrument and cross-player reactions,

\(^7\) Prior researchers have not distinguished between manufacturer and retailer reactions because they typically worked only with retail level data.
embodies an extrapolation of past interactions in that it assumes competitors and the retailer will continue to react to P&G and to each other in the wake of value pricing the same way they did before P&G’s move.

3.2 Non-Strategic Retailer

This benchmark incorporates the dynamic effects in our full game-theoretic model, with the exception that the retailer is non-strategic. It is assumed that the retailer will add a constant mark-up to the wholesale price and pass-through a constant percentage of the wholesale trade deal amount with no forward buying. The national brand competitor makes wholesale price and deal decisions in response to P&G’s actions, taking into consideration consumers’ immediate and dynamic response, but not the retailer’s. Therefore, this benchmark model embodies the possibility that national brand competitors do not engage in strategic competitive reasoning – they react to P&G but to not consider possible retailer reactions to their own decisions (Montgomery, Moore, and Urbany 2003).

4. EMPIRICAL ANALYSIS

4.1 Data

We use weekly retail level price, deal, and sales data from stores in the Chicago market belonging to the Dominicks grocery store chain, and we obtain weekly wholesale price and trade deal data in the Chicago market for P&G and other national brands from Leemis Marketing Inc., a company that specializes in tracking wholesale prices and trade deals in several U.S. markets. The Dominicks database provides a gross margin figure that can also be used to calculate “wholesale price”. However, as is cautioned in the documentation provided with the database, that calculation provides not the actual wholesale price in a given period but the average acquisition cost of the items in inventory, which is heavily influenced by the extent to which the retailer forward buys on deal. Similarly, the Dominicks database does not provide wholesale trade deal information. It can only be inferred from “dips” in the average acquisition cost but
these dips are also heavily influenced by forward-buying. The *Leemis* data provide information on actual wholesale price and wholesale trade deal amounts.

Like Ailawadi, Lehmann, and Neslin (2001), our analysis is at the level of a manufacturer in a product category. We aggregate the UPCs for a manufacturer up to one “umbrella brand.” Studying response at the umbrella brand level is consistent with packaged goods manufacturers’ emphasis on category management and with the strategic nature of the managerial situation. Value pricing was a company wide policy change made by P&G across the board and the response to this type of far-reaching policy change could also be expected to be strategic and broad-based. While most research in the sales promotion area aggregates over UPC’s (for instance, Chintagunta 1993; Ailawadi and Neslin 1999; and Silva-Risso, Bucklin, and Morrison 1999), we recognize that this raises the question of aggregation bias in demand function estimates if UPCs with heterogeneous prices and promotion are aggregated (Christen et al. 1997; Link 1995). To examine the impact on our results, we repeated our analysis by aggregating only those UPCs of each manufacturer whose retail prices are strongly positively correlated (e.g., Besanko, Dube, and Gupta 2003). There was no substantive change in our findings.

We were able to compile data for nine product categories in which P&G is a player. *Dominicks* sold a private label brand in six of these nine categories throughout the period of our analysis. The number of brands for which both wholesale and retail data are available varies across categories, ranging from three to six. In total, we have 43 brands across the nine categories (nine P&G brands, 28 national brand competitors, and six private label competitors).

*P&G* announced its value pricing strategy at the end of 1991 (Shapiro 1992) and implemented it over the next few years. The company’s annual reports during the period from 1992-93 to 1994-95 explicitly discuss the company’s move to value pricing. They state that, by
1993, 90% of the company’s brands were under this program, the process of reducing prices and cutting costs continued in 1994, and value pricing was a fundamental strategy at P&G as of 1994-95. Thus, it appears reasonable to assume that P&G’s actions were exogenous at least until 1994. Figure 1 further supports this assumption. It shows the median change in P&G’s pricing and dealing over the 1991-1996 period in the Chicago market.\(^8\) The sustained pattern till 1994 is consistent with exogeneity of these actions, but after 1994, there is a change in the dealing pattern that may suggest a modification of strategy based on market reaction. Interestingly, there is no mention of value pricing in the company’s annual report for 1995-96. We therefore study the changes made from 1991 to 1994 to be sure that the assumption of P&G exogeneity is reasonable and yet be able to cover the bulk of the changes made under value pricing.

<Insert Table 1 About Here>

As the timeline summarized in Table 1 shows, we use retail data from the first 37 weeks of 1991 to estimate our demand functions and reaction functions, and changes between the last 15 weeks of 1991 and the last 15 weeks of 1994 to test our predictions. Thus, there is a clean separation between the estimation and prediction portions of our analysis, in that the same data are not used for estimation and for prediction. There are two reasons for using a 15-week horizon in our optimization. First, our interviews with grocery retailers suggest that this reflects reality in that manufacturers and retailers do tend to look ahead one quarter when they make pricing and promotion decisions. In fact, a typical promotion planning calendar is for a quarter. Second, given that we have to run our model twice (1991 and 1994) for each brand, we need to keep the computational burden reasonable. For example, with a 15 week horizon each run takes about 17 hours on a dedicated UNIX machine, while a 25 week horizon takes over 48 hours.

<Insert Table 2 About Here>

Table 2 provides definitions and sources of all the variables used in the analysis. Three points deserve mention. First, we obtain estimates of the variable cost of brands in each category

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\(^8\) The pattern is similar for the full year in the Chicago market and also in the national market.
using *Dominicks* gross margin on its private label in each category. If the retailer produces its own private label or buys it in a perfectly competitive supplier market, we can assume the cost of the private label to the retailer is the variable cost. We compute this variable cost using information on the retail price and retail gross margin on the private label, and apply it to all brands in a category. Second, we assume that a brand’s unit holding cost per month is 5% of its wholesale price. This figure is based on our conversations with managers at four large Northeastern retailers. Third, we assume that, in the weeks when the retailer orders under a trade deal, it offers 80% of the trade deal to consumers. This is based on Silva-Risso, Bucklin, and Morrison (1999), who note that 80% is a typical rate according to both the manufacturer and the retailer they worked with. It is also in the ballpark of the 62% figure reported by food retailers in the 2001 Cannondale survey (Cannondale 2001). We reiterate, though, that this does not mean pass-through is fixed in our model. Since the retailer’s decision to forward-buy is endogenous, effective pass-through, i.e., what the retailer sells on deal to consumers relative to what he buys on deal from the manufacturer (e.g., Besanko, Dube, and Gupta 2003), is also endogenous. Still, we tested the sensitivity of our results to the rate by re-doing all of our analysis with 120% instead of 80%, and found that the predictive ability of our model remained unchanged. Bulow and Pfleiderer (1983) and Tyagi (1999) show that with a linear or concave demand function, the optimal pass-through rate is less than 100%, whereas with some convex demand functions, it is greater than 100%. Although their models neither differentiate between regular and promotional price cuts nor incorporate dynamics or forward buying, we thought it would be helpful to conduct our analysis with rates both less than and greater than 100%.

### 4.2 Demand Function Estimation

As Equation (1) shows, we estimate a separate demand function for each brand in each category. Since data are pooled across stores, we include a set of store-specific dummy variables to account for differences in the base level of sales (e.g., Kopalle et al. 1999). The model is estimated using all available data from each Dominicks store in the first 37 weeks of 1991. The
first five weeks are used to initialize the cumulative dealing variable \((CD)\) and create lags for the price and deal variables (Table 1). We employ 2SLS to control for possible endogeneity in price and deal variables, using the five weekly lags of the price and deal variables as instruments (e.g., Kadiyali et al. 2000), and do a grid search to find the value of the smoothing parameter \(\lambda\) that maximizes \(R^2\) for each brand.\(^9\) We constrain the own-price parameter to be non-positive and the own deal parameter to be non-negative to avoid unstable coefficient signs due to possible multicollinearity. This is in the spirit of Blattberg and George (1991), who advocate shrinkage estimation, which is a more sophisticated procedure but accomplishes the same task of avoiding “incorrect signs.” Note that wrong signs of price and deal coefficients would lead to problems in the optimization model that is calibrated using these coefficients. For instance, the highest possible price would be charged or a deal would never be offered.

### 4.3 Prediction Procedure

We compute the optimal values of the decision variables for the last 15 weeks of 1991, at which time P&G had not yet implemented value pricing, and for the last 15 weeks of 1994, by which time value pricing had been implemented. We ignore the last two weeks to alleviate end game effects, compute the average optimal values for the remaining 13 week periods in both 1991 and 1994, and subtract these averages. That difference is represented as \(\text{PREDCHG}_{ij}\), the predicted change in decision variable \(i\) for brand \(j\). This is compared to \(\text{ACTCHG}_{ij}\), the actual change in variable \(i\) for brand \(j\) similarly computed over the same 13-week periods in 1991 and 1994. Note that our objective is not to predict week-to-week changes but to predict broader changes over time in response to the value pricing move. That is why we average the weekly predictions in 1991 and 1994 and examine the change in those averages instead of examining the predictions in each week.

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\(^9\) Models in the new empirical industrial organization (NEIO) paradigm often specify and estimate both supply side and demand side models. However, it is not necessary to estimate the supply side in order to control for endogeneity in the demand side (e.g., Chintagunta, Bonfrer, and Song 2002). We thank Vrinda Kadiyali for helping us to sort out this issue.
For our dynamic optimization model and the non-strategic retailer benchmark, initial
values of the cumulative dealing variable (CD) are based on the previous quarter of 1991.\textsuperscript{10} We
then plug into the dynamic optimization the demand parameters and the actual values of P&G’s
wholesale price and deal amount during the last 15 weeks of 1991 and obtain optimal values of
each brand’s decision variables during this 15-week period. This is done via a wide grid search
around average values of all the decision variables to find values that maximize the sum of the
relevant player’s profits over the 15 weeks. We repeat the process for the last 15 weeks of 1994,
using initial values of the CD variable from the previous quarter of 1994.

For the reaction function benchmark model, initial values of all variables are based on the
previous quarter of 1991. We then plug in actual values of P&G’s wholesale price and deal
amount for the last 15 weeks of 1991, and get predicted values of all the decision variables for
each week. Predicted values of each decision variable in one week are used to update the long-
term smoothed value for the next week. We repeat the same process for the last 15 weeks of
1994. Note that since the reaction function predictions are unbounded and can go below zero as
well as to very high positive values, while predictions from the other two models are bounded by
the width of the grid search, we truncate the reaction function predictions to the same bounds.\textsuperscript{11}

In addition to the predictions from the game theoretic model and the two benchmarks, we
consider the predictive ability of some covariates. Multi-market contact and firm size are two
key brand characteristics that determine competitive reaction. Multi-market contact
(MULTIMKT) may increase competitive rivalry (Porter 1980) or lead to mutual forbearance
because the competing firms have a high stake in many shared markets (Bernheim and Whinston
1990; Shankar 1999). Small or fringe firms (SMALL) are expected to react differently from

\textsuperscript{10} Since the retailer is assumed to order in the first period, the only initial state variable we need is cumulative
dealing.

\textsuperscript{11} Truncating negative and unreasonably high values can only help the reaction function benchmark’s predictions.
We also tested its predictive validity without the truncation and found that, as expected, it was worse.
larger competitors, although it is not clear what the differences are (Spiller and Favaro 1984; Putsis and Dhar 1998; Shankar 1999). There may also be differences in reaction across categories, over and above the impact through different sales response elasticities. We include three category characteristics -- average category advertising (CAD), average category dealing (CDL), and average category purchase cycle (CPC) -- that other researchers have used in cross-category empirical analysis (e.g., Narasimhan, Neslin, and Sen 1996; Ailawadi, Lehmann, and Neslin 2001). Thus, the regression model we estimate for each decision variable i using each predictive model m is:

\[ ACTCHG_{ij} = \beta_0 + \beta_1 PREDCHG_{mj} + \beta_2 CAD_c + \beta_3 CDL_c + \beta_4 CPC_c + \beta_5 SMALL_j + \beta_6 MULTIMKT_j + \epsilon_{ij} \]  

(17)

5. RESULTS

5.1 Estimates of Brand Sales Model

In order to conserve space, and because the magnitudes of the parameters in the linear brand sales model would be difficult to interpret, we do not report the model estimates. Instead, we compute own price elasticities and own deal elasticities (both the current deal elasticity and the dynamic elasticity including the interaction effect of past dealing) for each brand at average values of its sales, price, deal amount, and long-term smoothed dealing. We also compute the cross price and cross deal elasticities for each of P&G’s competitors with respect to P&G’s price and deal amount since this cross elasticity depicts how much the brand’s demand is affected by P&G’s policy change.

<Insert Table 3 About Here>

Table 3 summarizes the median values of these five elasticities for each category as well as for the overall sample. There is substantial variation in both own and cross elasticities across brands and categories, which would lead to differences in the response predicted for the different

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12 We did not use dummy variables for the nine categories because they would control for category differences but not provide any insights into the source of those differences, and also because of the small number of observations we have in each regression especially for the benchmark models.
brands. The magnitudes and signs of the price elasticities are generally consistent with regular price elasticities reported in previous research. Some of the cross price and cross deal elasticities are not of the expected sign, but this is not unusual in empirical analyses and could be due to promotions and pricing growing the category at an individual store level, thus benefiting other brands in that store (van Heerde, Gupta, and Wittink 2003). As expected, total dynamic deal elasticity is smaller than current deal elasticity due to the negative interactive effect of past dealing.

The deal elasticities look small at first glance. However, note that elasticities typically reported in the literature measure the percentage increase in sales when the brand is on deal versus off. Our deal elasticity measures the percentage sales increase due to a 1% increase in deal amount. We use our model parameters to calculate the average predicted bump in sales for each brand due to a deal of 15% off regular retail price. The median bump across our sample is 216% if we consider only the positive effect of the current deal, which is typical of the kind of product categories we are studying. The median bump is 156% if we also consider the negative dynamic effect of past dealing, showing that dynamics are indeed quite important in assessing the effects of promotion.

5.2 P&G and Competitor Actions

Table 4 summarizes the actual changes taken by P&G, its national brand competitors, and the retailer in the Chicago market between the last 15 weeks of 1991 and of 1994. The median change in P&G’s wholesale price was -4%. This is perhaps not as strong a reduction as we might have expected given the publicized EDLP nature of P&G’s strategy. However, changes in wholesale price go directly against P&G’s profit margin. Consistent with value pricing, P&G decreased its dealing drastically in the Chicago market -- the median cut across categories is about 90%.

<Insert Table 4 About Here>

13 The pattern of changes looks similar even if we consider the entire year instead of just the last 15 weeks.
The table also shows that national brand competitors followed P&G’s lead on wholesale price by keeping it almost unchanged (-0.2%), but did not follow P&G’s lead on deals. They increased their wholesale dealing 150%. Thus, overall, competitors reacted aggressively on deals. Finally, Dominicks increased retail prices for P&G by about 7% and increased retail prices for competing national brands somewhat less (4.6%). Interestingly, the retailer increased the retail price of his private label very slightly (1.9%) but decreased dealing for the private label by almost 35%. In light of Ailawadi, Neslin, and Gedenk’s (2001) findings regarding value conscious consumers, the retailer’s actions can be interpreted as an accommodating move versus the national brands. In particular, those authors find that core store brand buyers are less interested in stockpiling and planning their shopping around deals. They recommend that store brands should be priced EDLP to satisfy this core customer group instead of reaching for the market segment that will buy national brands on deal or store brands, if the retailer wants to avoid head-to-head competition.

5.3 Predicting the Direction of Change

The game theoretic model and the reaction functions produce wholesale price and wholesale deal predictions for the 28 national brands; retail price predictions for the 28 national brands, the P&G brand in each of the nine categories, and the private label in the six categories where a private label was sold throughout the period of analysis; and retail deal predictions for the private label in those six categories. Since we do not have enough observations to estimate a private label deal regression separately, we include those observations in the wholesale deal regression. The non-strategic retailer benchmark model only makes wholesale price and deal predictions for the 28 national brands.

Table 5 summarizes the percentage of brands for which each model is able to make “admissible” predictions of change, i.e., the percentage of brands for which our optimal values in both 1991 and 1994 do not hit bounds. It also shows the percentage of admissible predictions.
where the direction of change predicted is in line with the direction of the actual change. The table highlights two important points. First, the percentage of predictions that are admissible is much higher for the game theoretic model than for either of the two benchmarks although we use the same wide bounds for all three models. The non-strategic retailer model performs particularly poorly, with only about half the predictions being admissible. Second, our model’s ability to predict the direction of change for all the decision variables is better than chance and substantially higher than the two benchmark models.

Figure 2 elaborates on Table 5 by comparing the ability of the three models to predict specific directions of change. For example, the figure shows that of the times when our game theoretic model predicted that competitors would increase wholesale price, competitors actually increased price 78% of the time. This compares with correct predictions of 25% for the reaction function approach and 36% for the non-strategic retailer approach. Figure 2 shows that the game theoretic model provides superior up and down predictions. The only exception is when it is out-predicted by the non-strategic retailer approach in the case of an up-prediction of wholesale deal amount. However, Table 5 shows that in this instance, the non-strategic retailer model could make a admissible prediction only 46% of the time, compared to 91% for the game theoretic model. So, our model was able to make twice the predictions, and achieved an accuracy of 74%, compared to the non-strategic retailer approach achieving 80% but with only half the predictions.

Overall, Table 5 and Figure 2 show that our game theoretic model (1) makes more admissible predictions than the reaction function or non-strategic retailer benchmarks, (2) correctly predicts the direction of change more often than the benchmarks, and (3) correctly predicts the specific direction of change (both up and down) more often than the benchmarks.

5.4 Predicting the Magnitude of Change

Table 6 provides the results of a bivariate regression of ACTCHG on PREDCHG for each decision variable and each model. In each regression, we use only those observations for
which the model is able to make admissible predictions. The table shows that the magnitude of
the actual change is significantly correlated with the magnitude of change predicted by the game
theoretic model for all three variables. In contrast, the correlation with the magnitude of change
predicted by the benchmark models is statistically significant only for the deal amount variable,
and the level of significance is weaker, particularly for the non-strategic retailer benchmark.

<Insert Table 6 About Here>

Table 7 provides descriptive statistics of the variables used in the multiple regression
model of equation (17). Table 8 provides estimates of the regression using the game theoretic
model predictions, and Table 9 provides estimates using the benchmark model predictions.14

<Insert Tables 7 and 8 About Here>

Table 8 reaffirms that the regressions using the game theoretic model predictions fit well,
with the adjusted $R^2$ for the three regressions ranging from 0.22 to 0.53. More importantly, the
coefficient of PREDCHG is positive and statistically significant for all three regressions. In fact,
as the standardized coefficients in the table show, PREDCHG is the most important predictor of
actual changes. This is an encouraging result. Using parameter estimates from an initialization
period and the stimulus provided by P&G’s major policy change, we computed the optimal
changes that competing national brands and the retailer should have made in response to P&G’s
move, and these regressions show that the actual changes made by these agents are significantly
associated with the optimal changes prescribed by the model.

A couple of the covariates also predict actual changes. Competitors appear to be
somewhat accommodating if they have multi-market contact with P&G -- they are less likely to
increase wholesale deal amounts and less likely to decrease wholesale price. This is consistent
with Bernheim and Whinston (1990) and Shankar (1999) who argue that multi-market contact
dampens rivalry. On the other hand, competitors tend to respond more aggressively in categories

14 PREDCHG is measured with error so our results may understate the true relationship between each model’s
prescriptions and actual response.
with longer purchase cycles – they are more likely to increase wholesale deal amounts and more likely to decrease wholesale prices. The other covariates do not directly affect actual response, over and above any indirect effect they may have through the demand parameters that in turn determine PREDCHG.

<Insert Table 9 About Here>

A comparison of Table 8 with Table 9 underscores the superior predictive ability of the game theoretic model relative to either benchmark. Among the five regression models using predictions from the benchmark models, only one has a significant overall F-statistic – the deal amount regression with predictions from the reaction function benchmark.

6. DISCUSSION

6.1 Summary of Findings

The primary conclusion of this research is that the prescriptions of a dynamic game theoretic model contribute significantly to the prediction of actual competitor and retailer response to a major policy change. We constructed our game theoretic model and “calibrated” it using an empirical analysis of sales data in an initialization period. We then substituted P&G’s price and dealing changes from a future period, and obtained optimal changes in the prices and dealing of national brand competitors and retailers. That the nature of optimal changes corresponded with the nature of actual changes is an important accomplishment, especially given the challenge in both formulating the dynamic structural model and estimating the dynamic demand functions. We did not “merely” regress response versus changes in marketing mix. We regressed response versus prescriptions of how competitors and retailers should respond to P&G’s actions, and found significant relationships.

It took a sophisticated dynamic game theoretic model to obtain predictions that were correlated with actual response. A reaction function based model that assumes competitors and the retailer will continue to react to each others decisions the same way they have in the past, even in the wake of a major policy shift, has some predictive ability but is clearly dominated by
our model. And a dynamic model that does not consider retailer response, assuming instead that the retailer will non-strategically apply a fixed price mark-up and wholesale deal pass-through rate, performs quite poorly.

The fact that this test is for a major policy change needs to be emphasized. Our findings support Montgomery, Moore, and Urbany’s (2003) view that strategic competitive thinking on the part of managers is more likely for highly visible and major decisions, especially in the context of pricing. It also supports the premise noted in the Introduction that structural models are preferable to reduced form models when the agents involved are adjusting to a change in regime. For more typical, ongoing decisions and week-to-week or month-to-month reactions, where managers are less likely to engage in competitive reasoning, the reaction function approach or a simplified game theoretic model may have better predictive ability.

It is also noteworthy that a couple of category and brand characteristics contribute to the prediction of competitor response. These characteristics already have an indirect influence through their effect on price and deal response parameters, which in turn drive the response predicted by our economic model. Therefore, a direct effect association with actual response over and above this is particularly interesting. Of particular note is the predictive ability of multi-market contact, a factor that descriptive research on incumbent response to new entrants has also identified. We find that national brand competitors who compete with P&G in multiple markets tend to be more accommodating. It is equally interesting that relative firm size does not have much predictive ability over and above the impact through response parameters.

6.2 Implications for Future Research

One limitation of our model is that we do not consider retail competition. In our case, we simply did not have data on competitive retailers, but including retailer competition would be a challenge (see Sudhir 2001). Another is that we assume the retailer provides a fixed percentage of the trade deal to consumers during weeks when it buys on trade deal. Although we do endogenize forward buying, which is a primary reason for observed variations in the “effective”
retail pass-through rate, i.e., the percent of product bought by the retailer on deal that is sold to the consumer on deal, it would also be worthwhile to make the retail deal amount endogenous. A third is that we assume deterministic demand. Although there is a strong tradition of deterministic models in the literature (e.g., Choi 1991; Kim and Staelin 1999; Kopalle, Rao, and Assunção 1996; McGuire and Staelin 1983; Moorthy 2001), introducing stochasticity would also be a valuable endeavor. Fourth, our model assumes exogeneity of P&G’s actions and we analyze a period in which this assumption is empirically supported. However, there is some evidence that P&G later started to modify its strategy in response to marketplace reaction. It would be interesting to model such a scenario. Finally, we only consider price and deal response, but competitors may also have reacted using other marketing instruments like advertising and coupons, on which we did not have data. Overcoming these limitations while keeping the complexity of the model within manageable limits is challenging but would be a very fruitful avenue for future research efforts.

Future research could also move beyond the Manufacturer-Stackelberg game that we have used in this paper, either by testing other channel interactions like Vertical Nash, or by utilizing the New Empirical Industrial Organization (NEIO) paradigm (e.g., Kadiyali et al. 2000; Putsis and Dhar 1998; VIlcassim, Kadiyali, and Chintagunta 1999). In the latter, an endogenous supply and demand side model is specified and the type of channel interaction is inferred from the signs and magnitudes of the estimated conduct parameters. Or, multiple games are formulated, corresponding sets of equations are estimated, and a determination is made as to which formulation is most consistent with the data. Perhaps researchers could characterize the type of game using the initialization period and then test its predictions in the 1992-1994 period. The benefit of such an approach is clear since it does not impose a particular game structure a priori. The challenge, however, would be to incorporate in this framework the dynamics that are so important in studying promotion.
6.3 Conclusion

Methodologically, our study shows that combining a game theoretic dynamic structural model with empirical estimates of demand functions can pay off in superior predictions of competitor response. The two crucial features of the model are dynamics (stockpiling, wearout, retailer forward buying, multi-period decision making) and multi-party decision-making (the Manufacturer-Stackelberg formulation). These factors make it difficult to obtain closed form solutions for normative models. But with increased computing power and better data, we are at the point where this is more feasible.

Substantively, our study demonstrates the benefits of examining significant policy changes. First, these are interesting managerially, but second, they are more likely to provide the exogenous change needed to measure competitor and retailer reactions. We believe this exogeneity is crucial. Consider the notion, advanced by Raju, Srinivasan, and Lal (1990), Narasimhan (1988), and others that promotions are mixed strategy equilibria. The implication is that, in equilibrium, manufacturers promote randomly and do not respond to each other on a week-by-week basis. This may explain why it is difficult to measure week-to-week competitor response in packaged goods markets. It takes a major policy change like Value Pricing – an out-of-equilibrium move in terms of Raju et al. (1990) and Narasimhan (1988) – to identify competitive response.

Managerially, the promise of our findings is that managers can use dynamic structural models to run scenarios to gauge competitive response. We say “promise” because our findings are initial, and we would advocate replications and extensions before managers can safely make major business decisions based on these predictions. However, our results suggest that empirically calibrated game theoretic models do have that potential.

Finally, this study analyzes but one instance of predicting competitive response to a major policy change – in the packaged goods industry when a “big player” makes a well-
publicized move. We need to generalize beyond this single instance to other industries and other circumstances. Our results provide encouraging evidence that this would be a fruitful endeavor.
### Table 1

**Timeline of Data**

<table>
<thead>
<tr>
<th>Year</th>
<th>Weeks</th>
<th>Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>1 to 5</td>
<td>Initialization of all long-term smoothed variables for demand functions and reaction functions</td>
</tr>
<tr>
<td></td>
<td>6 to 37</td>
<td>Estimation of demand functions and reaction functions</td>
</tr>
<tr>
<td></td>
<td>38 to 52</td>
<td>Optimization period before implementation of Value Pricing</td>
</tr>
<tr>
<td>1992</td>
<td>1 to 52</td>
<td>Value Pricing being implemented, data not used</td>
</tr>
<tr>
<td>1993</td>
<td>1 to 52</td>
<td>Value Pricing being implemented, data not used</td>
</tr>
<tr>
<td>1994</td>
<td>1 to 37</td>
<td>Value Pricing being implemented, data not used</td>
</tr>
<tr>
<td></td>
<td>38 to 52</td>
<td>Optimization period after implementation of Value Pricing</td>
</tr>
<tr>
<td>1995</td>
<td>1 to 52</td>
<td>Possible modification of P&amp;G strategy, data not used</td>
</tr>
<tr>
<td>1996</td>
<td>1 to 52</td>
<td>Possible modification of P&amp;G strategy, data not used</td>
</tr>
</tbody>
</table>
Table 2  
List of Variables and Definitions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wholesale price</td>
<td>Wholesale price per equivalent volume in a given week</td>
<td>Leemis Marketing</td>
</tr>
<tr>
<td>Wholesale deal amount</td>
<td>Wholesale dollar deal amount per equivalent volume in a given week</td>
<td>Leemis Marketing</td>
</tr>
</tbody>
</table>
| Retail price              | Regular retail price per equivalent volume in a given week:  
Equal to net price paid when brand is not on retail promotion, and equal to the maximum net price paid during the previous four weeks and next four weeks in weeks when brand is on retail promotion.  
Not all promotions are flagged in the Dominicks data. Following Gedenk and Neslin (2000), we flag a promotion in a given week if the net price paid is at least 5% lower than the maximum price paid in the previous four weeks and the subsequent four weeks. | Dominicks Data  |
| Retail deal amount        | Regular retail price minus net price paid.                                                                                                                                                                | Dominicks Data  |
| Unit sales                | Number of equivalent units sold in a given week.                                                                                                                                                           | Dominicks Data  |
| Variable cost             | Average value of:  
(1-% retail margin on private label) times (retail price of private label).                                                                                                                                 | Dominicks Data  |
| Size                      | Equals 1 if the market share of the brand is less than 5% of the total share of the top three brands in the category; equals 2 if it is between 5% and 40%; equals 3 if it is greater than 40%.                                      | Dominicks Data  |
| Multi-market contact      | Equals 1 if the firm owning the brand competes with P&G in more than two of the categories, 0 otherwise.                                                                                                    | Dominicks Data  |
| Category advertising      | Average annual media advertising expenditure in the category.                                                                                                                                             | Leading National Advertisers |
| Category dealing          | Average percentage of sales in the category made on some type of retail deal.                                                                                                                               | IRI Market Fact Book |
| Category purchase cycle   | Average number of days between consecutive purchases of the category.                                                                                                                                       | IRI Market Fact Book |

Note: Price and deal variables are averaged across all UPCs for a manufacturer.
Table 3

Summary of Brand Sales Model Estimates

<table>
<thead>
<tr>
<th>Category</th>
<th>Median Value of</th>
<th>Own Price Elasticity</th>
<th>Own Current Deal Elasticity</th>
<th>Dynamic Own Deal Elasticity</th>
<th>Cross Price Elasticity with P&amp;G</th>
<th>Cross Deal Elasticity with P&amp;G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Automatic Dishwasher Detergent Liquid</td>
<td></td>
<td>-10.783</td>
<td>0.067</td>
<td>0.044</td>
<td>10.230</td>
<td>-0.014</td>
</tr>
<tr>
<td>Automatic Dishwasher Detergent Powder</td>
<td></td>
<td>-0.206</td>
<td>0.138</td>
<td>0.109</td>
<td>3.988</td>
<td>-0.055</td>
</tr>
<tr>
<td>Dishwashing Liquid</td>
<td></td>
<td>-0.000</td>
<td>0.356</td>
<td>0.283</td>
<td>-1.080</td>
<td>-0.047</td>
</tr>
<tr>
<td>Fabric Softener Sheets</td>
<td></td>
<td>-1.950</td>
<td>0.154</td>
<td>0.112</td>
<td>1.533</td>
<td>-0.018</td>
</tr>
<tr>
<td>Laundry Detergent Liquid</td>
<td></td>
<td>-5.451</td>
<td>0.380</td>
<td>0.269</td>
<td>-0.344</td>
<td>-0.005</td>
</tr>
<tr>
<td>Laundry Detergent Powder</td>
<td></td>
<td>-1.740</td>
<td>0.170</td>
<td>0.153</td>
<td>-0.272</td>
<td>0.019</td>
</tr>
<tr>
<td>Paper Towels</td>
<td></td>
<td>-2.944</td>
<td>0.309</td>
<td>0.292</td>
<td>0.244</td>
<td>0.007</td>
</tr>
<tr>
<td>Toilet Tissue</td>
<td></td>
<td>-0.651</td>
<td>0.834</td>
<td>0.698</td>
<td>-0.435</td>
<td>-0.057</td>
</tr>
<tr>
<td>Toothpaste</td>
<td></td>
<td>-2.59</td>
<td>0.239</td>
<td>0.225</td>
<td>0.239</td>
<td>-0.027</td>
</tr>
</tbody>
</table>

Note: Elasticity for each brand is calculated at the mean values of sales and the independent variables. Dynamic deal elasticity is net of the interactive effect of smoothed past dealing.
### Table 4

**Actual Changes in the Chicago Market**

**Between Last Quarter of 1991 and Last Quarter of 1994**

<table>
<thead>
<tr>
<th>Category</th>
<th>% Change From Last Quarter 1991 to Last Quarter 1994</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Deal Amount</td>
</tr>
<tr>
<td>Auto. Dish Detergent Liquid</td>
<td>-100%  +1% --</td>
</tr>
<tr>
<td>Auto. Dish Detergent Powder</td>
<td>-67%  +2445% -67%</td>
</tr>
<tr>
<td>Laundry Detergent Liquid</td>
<td>-100%  +299% -74%</td>
</tr>
<tr>
<td>Laundry Detergent Powder</td>
<td>-94%  -63% -69%</td>
</tr>
<tr>
<td>Liquid Dish Detergent</td>
<td>-57%  +96% -76%</td>
</tr>
<tr>
<td>Paper Towels</td>
<td>-74%  -62% --</td>
</tr>
<tr>
<td>Sheet Fabric Softener</td>
<td>-77%  +135% --</td>
</tr>
<tr>
<td>Toilet Tissue</td>
<td>-94%  -59% -88%</td>
</tr>
<tr>
<td>Toothpaste</td>
<td>-91%  -69% +1000%</td>
</tr>
</tbody>
</table>
## Table 5

**Summary of Predictive Ability: Direction**

<table>
<thead>
<tr>
<th>Model</th>
<th>% Admissible Predicted Changes</th>
<th>% Directionally Correct Predicted Changes*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Deal Amount (n=34)</td>
<td>Wholesale Price (n=28)</td>
</tr>
<tr>
<td>Game theoretic model</td>
<td>91%</td>
<td>100%</td>
</tr>
<tr>
<td>Reaction function benchmark</td>
<td>68%</td>
<td>96%</td>
</tr>
<tr>
<td>Non-strategic retailer</td>
<td>46%</td>
<td>54%</td>
</tr>
</tbody>
</table>

* Computed as a % of admissible predicted changes
## Table 6

### Summary of Predictive Ability: Magnitudes

<table>
<thead>
<tr>
<th>Model</th>
<th>Regression of ACTCHG in</th>
<th>Deal Amount</th>
<th>Wholesale Price</th>
<th>Retail Price</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Game theoretic model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient of OPTCHG</td>
<td>0.308*** (5.38)</td>
<td>0.106*** (2.32)</td>
<td>0.097*** (3.88)</td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.508</td>
<td>0.183</td>
<td>0.295</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>30</td>
<td>26</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td><strong>Reaction function benchmark</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient of OPTCHG</td>
<td>0.244** (2.05)</td>
<td>-0.130 (-0.85)</td>
<td>0.001 (0.08)</td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.167</td>
<td>0.028</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>23</td>
<td>27</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td><strong>Non-strategic retailer benchmark</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient of OPTCHG</td>
<td>0.300* (1.58)</td>
<td>0.012 (0.09)</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.185</td>
<td>0.001</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>13</td>
<td>15</td>
<td>--</td>
<td></td>
</tr>
</tbody>
</table>

Note: t-stats are in parentheses.

***p<0.05; ** p<0.10; *p<0.15
### Table 7

**Descriptive Statistics**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Category Advertising (CAD)</td>
<td>$18.3 mill</td>
<td>$7.1 mill</td>
<td>$44.5 mill</td>
</tr>
<tr>
<td>Category Dealing (CDL)</td>
<td>38.9%</td>
<td>23.2%</td>
<td>55.9%</td>
</tr>
<tr>
<td>Category Purchase Cycle (CPC)</td>
<td>88.7 days</td>
<td>68 days</td>
<td>100 days</td>
</tr>
<tr>
<td>Small Share (SMALL)</td>
<td>0.19</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Multi-market Contact (MULTMKT)</td>
<td>0.38</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
### Table 8

**Game Theoretic Model Prediction Regression**

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Standardized Coefficient in Regression of ACTCHG in</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predicted Change (PREDCHG)</td>
<td>Deal Amount</td>
</tr>
<tr>
<td></td>
<td>0.657*** (4.97)</td>
</tr>
<tr>
<td>Category Advertising (CAD)</td>
<td>0.124 (0.94)</td>
</tr>
<tr>
<td>Category Dealing (CDL)</td>
<td>0.255 (0.90)</td>
</tr>
<tr>
<td>Category Purchase Cycle (CPC)</td>
<td>0.458* (1.62)</td>
</tr>
<tr>
<td>Small Share (SMALL)</td>
<td>0.180 (1.31)</td>
</tr>
<tr>
<td>Multi-market Contact (MULTMKT)</td>
<td>-0.250** (-1.76)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.628 (0.531)</td>
</tr>
<tr>
<td>F-stat</td>
<td>6.46***</td>
</tr>
</tbody>
</table>

| n                                              | 30                              | 26                   | 38                  |

Note: t-stats are in parentheses except in the $R^2$ row, where adjusted $R^2$’s are in parentheses.

$$*** p<0.05; \quad ** p<0.10; \quad * p<0.15$$
Table 9
Benchmark Models Prediction Regression

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Reaction Function Benchmark</th>
<th>Non-Strategic Retailer Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standardized Coefficient in ACTCHG Regression</td>
<td>Standardized Coefficient in ACTCHG Regression</td>
</tr>
<tr>
<td></td>
<td>Deal Amount</td>
<td>Wholesale Price</td>
</tr>
<tr>
<td>Predicted Change (PREDCHG)</td>
<td>0.404***</td>
<td>-0.353</td>
</tr>
<tr>
<td></td>
<td>(2.49)</td>
<td>(-1.27)</td>
</tr>
<tr>
<td>Category Advertising (CAD)</td>
<td>0.601***</td>
<td>0.219</td>
</tr>
<tr>
<td></td>
<td>(3.00)</td>
<td>(0.85)</td>
</tr>
<tr>
<td>Category Dealing (CDL)</td>
<td>-0.181</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>(-0.66)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Category Purchase Cycle (CPC)</td>
<td>0.567**</td>
<td>-0.014</td>
</tr>
<tr>
<td></td>
<td>(1.95)</td>
<td>(-0.03)</td>
</tr>
<tr>
<td>Small Share (SMALL)</td>
<td>-0.145</td>
<td>0.055</td>
</tr>
<tr>
<td></td>
<td>(-0.79)</td>
<td>(0.22)</td>
</tr>
<tr>
<td>Multi-market Contact (MULTMKT)</td>
<td>-0.253*</td>
<td>0.175</td>
</tr>
<tr>
<td></td>
<td>(-1.58)</td>
<td>(0.59)</td>
</tr>
<tr>
<td>R²</td>
<td>0.612</td>
<td>0.086</td>
</tr>
<tr>
<td></td>
<td>(0.467)</td>
<td>(-0.189)</td>
</tr>
<tr>
<td>F-stat</td>
<td>4.21***</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td></td>
<td>23</td>
</tr>
</tbody>
</table>

Note: t-stats are in parentheses except in the R² row, where adjusted R²’s are in parentheses.

***p<0.05; ** p<0.10;  *p<0.15
Figure 2

Comparison of Predictive Ability: Specific Direction of Change

Note: Bars represent the percent of time when the actual change was up (down), among cases where the predicted change was up (down). The base is the number of admissible predictions.
APPENDIX

A.1: Dynamic Programming Algorithm for Manufacturer and Retailer Decisions

Beginning of backward induction: For $t = T$ to 1 (where $V_{RT+1} = 0$ and $V_{MT+1} = 0$),

Beginning of loop for manufacturer 2’s state vector: For each possible value of the state vector, $[L_{2t}, WP_{L2t}, D_{L2t}, CD_{2t}, CD_{3t}]$, where $L_{2t}$ is the period of last order placed by the retailer for Brand 2, $WP_{L2t}$ and $D_{L2t}$ are the optimal wholesale price and trade dealing during the last purchase period, $CD_{2t}$ and $CD_{3t}$ are the cumulative retail amounts for Brands 2 and 3 until period $t$,

Beginning of loop for manufacturer 2’s decision variables: For each possible value of manufacturer 2’s decision variables, wholesale price, $WP_{2t}$, and trade deal amount $D_{2t}$ (note that $D_{2t} = 0$ is the case of no trade deal during period $t$),

Beginning of loop for retailer decision variables: For each of the two possible values of $I_{2t}$ (where $I_{2t} = 1$ if the retailer purchase Brand 2 in period $t$ and $I_{2t} = 0$ if the retailer carries forward from last purchase),
Determine optimal retail prices of Brands 1, 2, and 3, whether to offer a retail deal for the private label and if so how much, by computing the retail profit in period $t$ plus cumulative future maximum retail profit,

End of loop for retailer decision variables: Determine whether it is profitable for the retailer to buy now from Brand 2 or to carry forward inventory from past period $L_{2t}$ for Brand 2. Store the maximum retail profit as $\pi_{Rt}$ as a function of the state vector and manufacturer decision variables,

End of loop for manufacturer 2’s decision variables: Increment to next value of manufacturer 2’s decision variables, $WP_{2t}$ and $D_{2t}$, Determine optimal wholesale price for Brand 2 and whether to offer a trade deal for Brand 2 and if so how much, by computing the manufacturer profit in period $t$ plus future maximal manufacturer profit. Store the corresponding manufacturer and retailer optimal decisions, and the maximum profits for the manufacturer, $V_{Mt}$, and the retailer, $V_{Rt}$, by identifying $\pi_{Rt}^*$.

End of loop for the state vector: Increment to next value of the state vector $[L_{2t}, WP_{L2t}, D_{L2t}, CD_{2t}, CD_{3t}]$.

End of backward induction: Decrement to next value of $t$. 
A.2: Details Of The Manufacturer-Retailer Equilibrium

In the last period $T$, for a given set of values of the state vector and the manufacturer’s wholesale price and trade deal amount, we use grid search to determine optimal retail prices for the three brands and retail deal amount for the private label under each of the following scenarios: (1) retailer carries forward inventory of Brand 2 from period $L_2T$ to period $T$, i.e., $I_{2T} = 0$; and (2) retailer orders Brand 2 in period $T$, i.e., $I_{2T} = 1$. The ordering decision ($I^*_{2T}$) that produces highest total profit is chosen and that profit is denoted by $\pi^*_{RT} (L_{2T}, WP_{L2T}, D_{L2T}, CD_{2T}, CD_{3T}; WP_{2T}, D_{2T})$. For that set of values, the optimal retailer decisions in period $T$ are now embedded in the manufacturer’s backward induction algorithm. The manufacturer begins with the last period, $T$, where $V_{MT+1} = 0$. For a set of values for the state vector the manufacturer optimizes $WP_{2T}$ and $D_{2T}$ via a grid search, by taking into consideration retailer reactions in period $T$. The corresponding optimal values are given by $WP^*_{2T}$ and $D^*_{2T}$, and the maximum profit is $V^*_{MT}$. This process is iterated for all possible values of the state vector.

Next, we consider period $T-1$. Just as we did in period $T$, we determine the optimal retailer ordering decision ($I^*_{2,T-1}$) and the optimal retailer profit ($\pi^*_{RT-1}$) for a given set of values of the state vector and Brand 2’s wholesale price and trade deal amount. However, these variables are now optimized by considering the retailer’s and the manufacturer’s optimal decisions in period $T$ via $V_{RT}$, the retailer’s cumulative maximum future profit component of $\pi^*_{RT-1}$. Note that $V_{RT}$ is identified by the state variables at the beginning of period $T$ and is computed using equations (2) and (6). This process is now intertwined with the manufacturer’s decisions in period $T-1$. Next, the manufacturer then considers period $T-1$ and optimizes $WP_{2T-1}$ and $D_{2T-1}$ by considering (1) the corresponding retailer’s optimal decisions in period $T-1$, and (2) the retailer’s and the manufacturer’s optimal decisions in period $T$ via $V_{MT}$, the cumulative maximum future profit for the manufacturer. Note that $V_{MT}$ is identified by the values of the state variables at the beginning of period $T$, which are computed in the optimization process using equations (2) and (6). This process repeats for all possible values of the state vector and for each time period until the beginning of the problem horizon is reached.

When the backward induction algorithm is complete, a forward propagation algorithm determines the optimal path for the retailer and the manufacturer using the initial conditions for $CD_{21}$ and $CD_{31}$, past cumulative retail deal amounts for Brands 2 and 3 respectively. The retailer places an order in period 1. The values of the state vector in period 2 are obtained by combining the optimal decisions in period 1 with equations 2 and 6. This forward algorithm proceeds until the end of time horizon, $T$, is reached, thus identifying the equilibrium manufacturer and retailer decisions in periods 1 through $T$. Note that in our model, the number of periods for which the retailer forward buys Brand $i$ is a function of retailer costs for Brand $i$ as well as what the retailer expects the manufacturer to offer in the future with respect to wholesale prices and trade deal amounts. Thus, retailer forward buying, retail pricing decisions for the three brands, retail deal amount for the private label, Brand 2’s wholesale price and trade deal amount are all dynamic, interdependent, and endogenous decision variables.
REFERENCES


