Capital Markets

Performing Bond Calculations

The principles of present value can be directly applied to the valuation of bonds. Essentially, a bond is an obligation to make a fixed payment or series of payments of a designated amount at a designated time. Because the characteristics of the bond (maturity, coupon rate) dictate when each payment will occur and how much each payment will be, bonds can be quite precisely valued using discounted cash flow analysis (specifically, NPV and IRR).

Bond Basics

One can think of bonds as having two payment components: principal and interest. The terms below are used to describe what principal and interest are and how they behave.

Principal: Principal refers to the amount of money one loans to the bond issuer. The principal invested in a bond may be more or less than the bond's par value.

Par (Face) Value: For standard bonds, Par Value represents the amount of principal upon which interest payments are based as well as the amount of principal that will be returned to an investor when the bond matures.

Nominal Interest Rate: The Nominal Interest Rate is the stated "rate" of interest a bond pays per year (the nominal interest rate is also called the "coupon rate"). It is not really a "rate" but an amount of interest as a percentage ("rate") of the par value. However, interest payments are actually calculated using the periodic interest rate (see below).

Periodic Interest Rate: The interest rate per period, calculated by taking the nominal interest rate and dividing by the number of periods per year. For example, a semi-annual bond with a nominal interest rate of 8.00% has a periodic interest rate of 4.00%. Thus, each period the bond makes an interest payment of 4.00%. Virtually all government, corporate and municipal bonds use a semi-annual convention (i.e., make two interest payments per year).

Coupon/Interest Dates: The dates on which interest payments are made are referred to as *interest dates* or *coupon dates*. Typically, as mentioned above, interest is paid semi-annually and therefore there are typically two interest dates per year.

Interest Payment: (also called *coupon payment*) Interest payments are made each period (usually semi-annually) and are calculated by multiplying the periodic interest rate times the par value of the bond. For example, a $100,000 par value semi-annual bond with a 6% nominal interest rate would pay $100,000 x 3% = $3,000 every six months.

Maturity Date: The date on which principal is returned to the investor is called the maturity date. More generally, if one invests in a bond that repays its principal in five years, the bond is said to have a five-year maturity.

Price: Price can be thought of in two ways. First, the present value of the bond's cash flows represents the cash cost for the bond. Typically, the cash cost is expressed as a percentage of par such that a price of $99.00 implies a cash cost of 99% of par value. Bonds sold "at par" are quoted at $100.00, or 100% of par value. Note that the quoted price has nothing to do with how many bonds are represented, but rather it is a ratio of present value to par value.

Example: What is the present value of $1,500,000 par value bonds quoted at $101.50?
Answer: $1,500,000 x 101.50% = $1,522,500

Example: What is the present value of $500,000 par value bonds quoted at $88.75?
Answer: $500,000 x 88.75% = $442,500

Internal Rate of Return: (IRR, also called discount rate, or simply rate of return. IRR is similar to y*ield*, a term that will be introduced later in the course in much more detail.) For any given price, the series of cash flows that the bond provides represents a rate of return. That rate of return, or IRR, will increase as the price of the bond decreases, and decreases as the price of the bond increases. If someone offered to sell you the right to receive $1,000 three months from now, the rate of return represented by your decision to invest in that future cash flow would depend on the price you paid for it. The IRR would be higher if you paid $400 for it rather than if you paid $750 for it.

There are two fundamental concepts you should keep in mind when working with bonds:

Fundamental Concept One:

A bond represents a fixed series of payments that extends into the future. The value of that stream of payments varies depending upon the discount rate applied to it. Thus, the discount rate applied to a bond determines the bond's price. Conversely, if the price of a bond is known, then the discount rate can be calculated. Therefore, one can always calculate a price when given a discount rate and one can always calculate a discount rate if given a price.

Fundamental Concept Two:

Bonds priced at par have an internal rate of return (IRR) equal to their nominal interest rate (take a minute to think about why this is true). By similar logic, bonds priced at less that par ("discount bonds") have an IRR higher than the nominal interest rate. The increase in IRR stems from the fact that, for example, the holder of a discount bond pays $980 for a bond that will mature at $1000. It then follows that bonds priced greater than par ("premium bonds") have an IRR lower than the nominal interest rate because the bondholder "loses" any amount paid in excess of the maturity value of the bond.

The rest of this handout steps through several bond pricing exercises. It is intended to help you become familiar with how bonds work and how to solve bond related problems both with a financial calculator and with Excel. You must learn at least one method so you can find price from yield or yield from price quickly and efficiently.

Coupon Bonds

Consider fifty $100 par value bonds with a 7% coupon that pays semi-annually maturing in four years priced at $98.50.

Par Value	=	$100	
Nominal Interest Rate	=	7.00%	
Periodic Interest Rate	=	3.50%	7% nominal rate divided by two periods per year
Payment	=	$3.50	Par Value x Periodic Interest Rate
Coupon date	=	2/year	
Maturity	=	8 periods	Four years x two periods/year
Present Value	=	$98.50	$100.00 par value x 98.50%

Concept Check ^ *The bond is priced at a discount, therefore the IRR (or RATE) should be greater than the coupon rate*

With the information above, an IRR for the bond can be calculated using the RATE function in Excel:

	A	B	C	D	E
1	Price of Bond	=PV	$ (98.50)		
2	Par Value	=FV	$ 100.00		
3	Coupon Rate		7.00%		
4	Years to Maturity		4		
5	Payments/Yr		2		
6	No. of PMTs	=Nper	8	=C5*C4	
7	Interst Payment	=PMT	$ 3.50	=(C3/C5)*C2	
8					
9	Periodic Rate	=RATE	3.72%	=RATE(C6,C7,C1,C2)	
10	Nominal Rate		7.44%	=C5*C9	
11					
12					

Note that the RATE must be converted from periodic terms to nominal terms.

Conversely, if the discount rate of the bond is given as 6.75%, then the PV function can be used to determine the bond's price:

	A	B	C	D	E
1	Par Value	=FV	$ 100.00		
2	Coupon Rate		7.00%		
3	Years to Maturity		4		
4	Payments/Yr		2		
5	No. of PMTs	=Nper	8	=C5*C4	
6	Periodic Rate		3.50%	=C2/C4	
7	Payment	=PMT	$ 3.50	=C6*C1	
8	Discount Rate		6.75%		
9	Periodic Disc Rate	=RATE	3.38%	=C8/C4	
10					
11	Price of Bond	=PV	-$100.86	=PV(C9,C5,C7,C1)	
12	Percent of Par		-$1.0086	=C11/C1	
13	Quoted Price		$100.86	=C12*-100	
14					

The appropriate IRR for the bond is given as 6.75%, which is less than the nominal interest rate. Therefore, the bond should be priced at a premium, which is exactly what the calculation shows.

Remember that the quoted price is always a percentage of par value as seen in cells A12:D13.

The problem states that you are looking at fifty bonds. In both calculations, the present value of the fifty bonds can be calculated from the quoted price. Using the $98.50 price, the present value of the fifty bonds equals 50 x $98.50 = $4925.00. Using the $100.86 price, the present value of the fifty bonds equals 50 x $100.86 = $5043.19. Thus, it doesn't matter if you base your calculations on one $100 bond or all fifty (i.e., $5,000 par value) as long as you keep track of the correct PMT and Par Value.

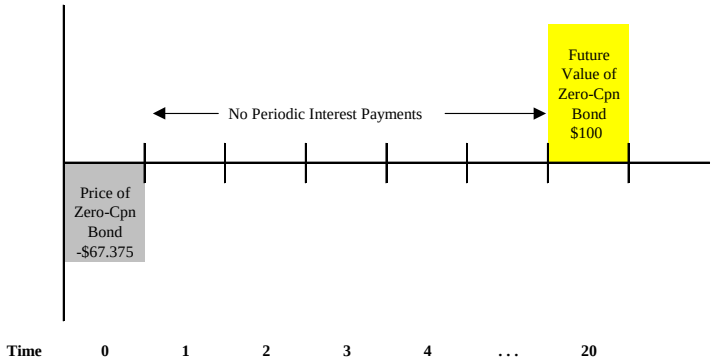
Note that both of the calculations shown above also can be executed on a financial calculator as well once the formula components (i.e., FV, Nper, PMT, and I% (RATE) or PV) have been determined.

Zero-Coupon Bonds

Zero-coupon bonds are perhaps the easiest to conceptualize. As the name implies, zero-coupon bonds do not pay periodic interest. Instead, the bonds are sold at deep discounts to par value. Thus, the interest rate is "built in" to the difference between the par value of the bond and its price.

Consider a $100 par value semi-annual zero-coupon bond that matures in 10 years and is priced at $67.375. What is the Internal Rate of Return (IRR) on the bond?

Step One: Diagram the cash flows



There are twenty periods (10 years, semi-annual) and no periodic interest payments are made.

Note that the future value of the bond equals its par value.

Step Two: Determine Formula and Formula Components

In this case, we are solving for the Interest Rate (*i*) that will solve a basic FV or PV equation:

$$FV = PV(1+i)^{Nper} \quad \text{OR} \quad PV = \frac{FV}{(1+i)^{Nper}}$$

PV	=	-$67.375
FV	=	$100.00
Nper	=	20

Step Three: Solve for Present Value

Using the PV equation

Plug and Chug:

Solve for *i*	=	0.01994

Note: Interest Rate is calculated using 20 semi-annual periods. Therefore, to express i as a nominal rate, it must be doubled. Thus, the interest rate = 2 x 0.1994 = 3.99%

Using Excel

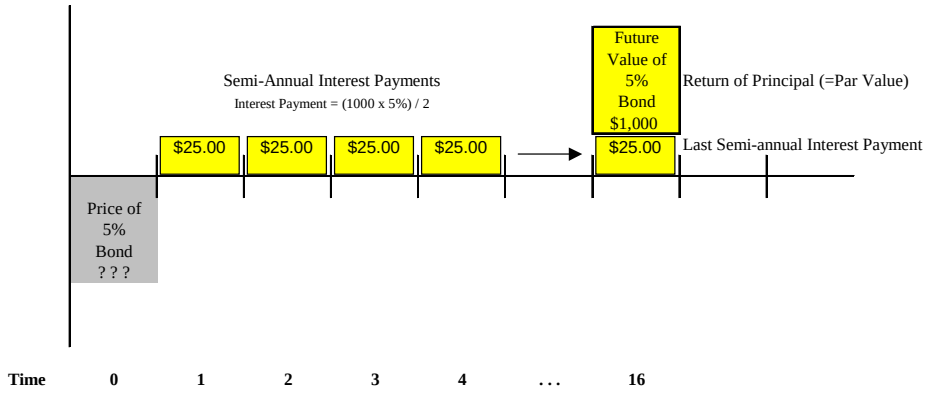
	A	B	C	D
1	PV	-67.375		
2	FV	100		
3	Nper	20		
4	I	0.019941		
5		=RATE(B3,,B1,B2)		
6				

Coupon Bonds Revisited

Bonds that pay periodic interest are called interest-bearing bonds or coupon bonds. Interest payments typically are made in semi-annual installments up to and including the day of maturity. On the maturity date, a bond makes its final coupon payment in addition to returning the par value or remaining principal balance.

Consider a $1000 bond with a 5% coupon that matures in 8 years. What would be the price of the bond if the appropriate discount rate were 5.85%?

Step One: Diagram the Cash Flows



Step Two: Determine Formula and Formula Components

The bond can be divided into two parts: principal and interest. The interest payments represent a fixed annuity stretching 16 periods into the future. The principal represents a single payment received in period 16. Therefore, one can combine the Annuity Formula and the basic PV formula to find the price of the bond.

$$PV = \frac{PMT}{i}\left[1-\left(\frac{1}{1+i}\right)^{Nper}\right] \quad \text{and} \quad PV = \frac{FV}{(1+i)^{Nper}}$$

PMT	= $25	($1000 x 5%) divided into semi-annual installments
Nper	= 16	8 years of semi-annual periods
FV	= $1000	The amount of principal returned at maturity = Par Value
i	= 5.85%	Remember *i* is the discount rate, not the coupon rate,
	= 2.925%	and must be divided into semi-annual periods

Step Three: Solve for PV

Using Formulas

Interest PV	= -$315.84
Principal PV	= -$630.47
Total Pv	= -$946.31

Using Excel or a financial calculator

	A	B	C
1	PMT	25	
2	Nper	16	
3	FV	1000	
4	I	0.02925	
5	PV	-946.3079	
6		=PV(B4,B2,B1,B3)	

Excel automatically combines the formulas shown above to perform one PV calculation.

Calculating discount rates using Internal Rate of Return (IRR)

Let's look at the previous example in a different way. Suppose that the present value of the bond was known to be $1025.00. How would you calculate the discount rate? Since the amount and timing of the cash flows are known, a basic IRR calculation will work. The diagram below illustrates how the calculation can be set-up and solved using Excel.

The quoted price for the bond equals the Present Value divided by Par Value, or 101.75% of par, which is quoted as a dollar price of $101.75

PV of Bond = $1017.50

Par Value	=	$1000.00
Coupon Rate	=	5.00%
Maturity Date	=	8 years
Payment Freq.	=	semi-annual
Payment	=	$25

	A	B	C	D	E
1	Price of Bond	=PV	$ (1,017.50)		
2	Par Value	=FV	$ 1,000.00		
3	Coupon Rate		5.00%		
4	Years to Maturity		8		
5	Payments/Yr		2		
6	No. of PMTs	=Nper	16	=C5*C4	
7	Payment	=PMT	25	=(C3/C5)*C2	
8	Internal Rate of Return	=IRR	2.367%	=IRR(B12:B28)	
9	Annual Rate of Return	= 2 x IRR	4.735%		
10					
11	Period	Cash Flow			
12	0	-1,017.50			
13	1	25.00			
14	2	25.00			
15	3	25.00			
16	4	25.00			
17	5	25.00			
18	6	25.00			
19	7	25.00			
20	8	25.00			
21	9	25.00			
22	10	25.00			
23	11	25.00			
24	12	25.00			
25	13	25.00			
26	14	25.00			
27	15	25.00			
28	16	1,025.00			

The IRR calculated by Excel is a periodic rate. Therefore, the IRR must be doubled to be expressed in nominal terms as shown in cells A9:C9.

Note that the cash flows contained in cells B13:B28 match the cash flow diagram on the previous page.

Note also that because each cash flow is listed separately, the IRR calculation can be performed when the payments are uneven or intermittent.

This IRR calculation can also be performed using a financial calculator. Use the following steps to solve the above example (for HP 19BII and similar calculators):

[FIN]	Puts HP 19BII calculator in financial mode
[CFLO]	Goes to Cash Flow functions
[v] [CLEAR DATA] [YES]	Clears memory
1020.50 [+/-] [INPUT]	Inputs the initial payment (negative for cash out flow)
25 [INPUT]	Inputs the interest payment
15 [INPUT]	Tells calculator to repeat interest payment 15 times
1025 [INPUT]	Inputs the final principal + interest payment
[CALC]	Goes to calculation menu
[IRR%]	Tells calculator to perform IRR calculation
= 2.367322	Answer is displayed

NOTE: The IRR% is a periodic rate that must be converted to a nominal rate (= 4.734664%)

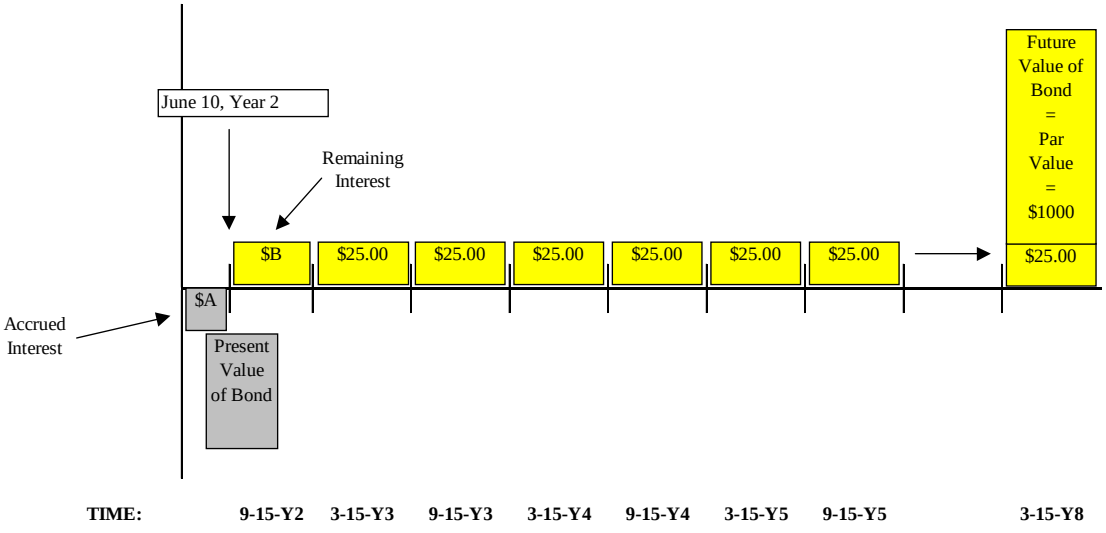
Accrued Interest

Imagine that you own a bond that pays interest semi-annually on March 15 and September 15. What would you do if you had to sell the bond on June 10? The next coupon doesn't pay until September – do you simply forfeit the interest you have earned through June 10? The answer, of course, is no, you don't forfeit any interest. Instead, you are entitled to receive an *accrued interest* payment from whoever purchases the bond from you. Thus, when you purchase a bond you must pay for both principal (i.e., the present value of the bond's future cash flows) AND accrued interest (i.e., the amount of interest that is rightfully due to the bond's seller).

Now that you are assured that you will receive the proper amount of interest, how do you price the bond? Heretofore the PV calculations we have performed were based on evenly spaced payments, but now the first interest period (June 10 to September 15) is shorter than all the others. Let's look at an example of how to account for accrued interest. Assume that the bond described in the previous example pays interest on March 15 and September 15 and that the bond matures on 3-15-Year 8. Today is June 10, Year 2 and you want to exchange the bond for cash (i.e., you exchange the bond for cash today). What is the price of the bond (principal and accrued interest) using an IRR of 4.90% (ignore the price calculated in the previous example)?

Concept Check: *Is this a par bond, a discount bond or a premium bond?*
The IRR is less than the coupon rate. The only way one could have a rate of return lower than the coupon rate is if a premium is paid for the bond (or the bond defaults, which is a topic for another day).

Before performing any calculations, it is helpful to conceptualize amount and timing of the cash flows. A diagram of the cash flows looks like this:



Essentially, this problem is the same as the previous example except that the first coupon period is divided into two parts. All the interest on the bond earned after June 10 goes to the purchaser, while you are entitled to the interest earned on the bond from the date of the last payment until the date you sell the bond.

In the Treasury markets, the convention for counting days is to count actual days in the year in question. Most corporate and municipal bonds assume for simplicity that each month has thirty days (this is called the 30-day convention or 30/360 basis). Since this bond pays semi-annually, each interest payment is pro-rated over a 180-day period such that the daily interest accruing on our $1000 par value 5% bond equals

$0.1389.[1] There are 85 days from the date of the previous coupon to the date of sale (March 15 to June 10) and 95 days from the date of sale to the date of the next coupon (June 11 - September 15) using a 30/360 basis. Thus, the accrued interest on the bond = (25.00/180) x 85 = $11.806.

It can further be seen that if 180 days equals one period, then time between June 10 and September 15 can be represented as a fractional period = 95/180 = 0.5278. Thus the first period in our PV calculation is fractional, and each period thereafter increases by one (see Excel illustration).

Using Excel

Here the accrued interest calculation is modeled in Excel:

	A	B	C	D	E	F
1						
2			Date	Day Count	% of Period	
3	Previous Coupon		3/15/Y2			
4	Today		6/10/Y2	85	47.2%	
5	Next Coupon		9/15/Y2	95	52.8%	
6	Total Days (30 day convention)			180	100.0%	
7						
8	Nominal IRR		4.90%			
9	Periodic IRR		2.45%	=C8/2		
10						
11	**PERIOD**	**DATE**	**PMT**	**PV**		
12	Last PMT	3-15-Y2	25.00			
13	0.00	6-10-Y2	**11.806**	**1,004.894**	=SUM(D14:D25)-C13	
14	0.53	9-15-Y2	25.00	24.68	=C14/(1+$C$9)^A14	
15	1.53	3-15-Y3	25.00	24.09		
16	2.53	9-15-Y3	25.00	23.52		
17	3.53	3-15-Y4	25.00	22.95		
18	4.53	9-15-Y4	25.00	22.40		
19	5.53	3-15-Y5	25.00	21.87		
20	6.53	9-15-Y5	25.00	21.35		
21	7.53	3-15-Y6	25.00	20.84		
22	8.53	9-15-Y6	25.00	20.34		
23	9.53	3-15-Y7	25.00	19.85		
24	10.53	9-15-Y7	25.00	19.38		
25	11.53	3-15-Y8	1,025.00	775.43	=C25/(1+$C$9)^A25	
26						

Note that the price (present value) of the bond equals the sum of the present values of the cash flows minus accrued interest.

Concept Check: *What is the quoted price of the bond?*

=Present Value / Par Value	=	$1004.894 / $1000.00
=100.489% of par	=	$100.489

[1] ($1,000 par x (5% semi-annual, or 2.5%))/180 = PMT/180 = 25/180 = .138889

Using a Calculator

This problem is not easy on a calculator unless your calculator has "bond math" built into it. If it does have bond math, then what the calculator does (behind the scene) is to solve for the bond's Price after first calculating the Accrued Interest.

Using bond functions in Excel

Excel has a number of functions that can be used to price bonds. This section covers only the very basic functions. More advanced functions and applications will be introduced later in the course.

Let's review the bond from the previous example:

Par Value	=FV	=	$1000
Present Value	=PV	=	$1020.50
Coupon		=	5.00%
Coupon Payment	=PMT	=	$25.00
Discount Rate	=I% or IRR	=	4.90%
Maturity		=	3-15-Y8
Today (sale date)		=	6-10-Y2
Previous Coupon Date		=	3-15-Y2
Next Coupon Date		=	9-15-Y2

The bond functions in Excel require the use of actual days and dates. That is to say that Excel bond functions uses dates instead of period numbers. To use the Excel functions, we have to change our fictional dates to dates in the calendar system. Therefore, let today, 6-10-Y2, become 6-10-2002 and so on. The new maturity date becomes 3-15-2008. Here is how Excel calculates the price and accrued interest in the above example using a nominal IRR (also called *yield*) of 4.90%:

	A	B	C	D	E	F
1	Sale Date (today)	=Settlement	06/10/02			
2	Maturity	=Maturity	03/15/08			
3	Previous Coupon Date	=Issue	03/15/02			
4	Next Coupon Date	=First_interest	09/15/02			
5	Coupon	=Rate	5.00%			
6	IRR	=Yield	4.90%			
7	Future Value	=Redemption	$ 100.00			
8	Coupons/year	=Frequency	2			
9						
10						
11	PRICE		$ 100.489	**=PRICE(C1,C2,C5,C6,C7,C8)**		
12	ACCRUED INTEREST		$ 11.806	**=ACCRINT(C3,C4,C1,C5,1000,C8)**		
13						
14	Present Value of Bond	=1000 x 100.49%	$ 1,004.89			
15	+ Accrued Interest	(from above)	$ 11.81			
16	Cash Cost of Bond		$ 1,016.699			
17						

Excel can also be used to calculate the IRR or Yield of a bond if the price is known. Using our same bond, find the yield if the price were known to be $101.25.

	A	B	C	D	E	F
1	Today	=Settlement	06/10/02			
2	Maturity	=Maturity	03/15/08			
3	Previous Coupon Date	=Issue	03/15/02			
4	Next Coupon Date	=First_interest	09/15/02			
5	Coupon	=Rate	5.00%			
6	Quoted Price	**=Par**	$101.25			
7	Future Value	=Redemption	$100.00			
8	Coupons/year	=Frequency	2			
9	Total Par Value		$1,000.00			
10						
11	IRR	=Yield	4.75%	**=YIELD(C1,C2,C5,C6,100,C8)**		
12	ACCRUED INTEREST		$11.81	**=ACCRINT(C3,C4,C1,C5,1000,+C8)**		
13						

Notice that the YIELD calculation refers to the quoted price as "Par." That nuance can cause confusion, so it is important to understand how different terms are used in different financial calculators.

When performing bond calculations either with Excel or with a financial calculator it is good to be able to apply some common sense tests to make sure the answer you get makes sense. In the example above, we know that the IRR or Yield of the bond must be less than the coupon rate because the price is greater than $100.00. Since we can think of the premium paid (or, conversely, a discount) for a bond as being amortized over the length of time to maturity, we can make some generalizations about premiums and discounts. A large premium or discount with a short time until maturity implies that the IRR or Yield will be much different than the coupon rate. Alternatively, one can surmise that a smaller premium or discount with a long time until maturity will have an IRR or Yield closer to the coupon rate. The bond above passes this quick mental test: there is only .25% difference between IRR and coupon for about a 1.25% premium over six years. Being able to "eyeball" calculations like this can be a great help, especially when struggling with unwieldy financial calculators.

More bond functions in Excel will be introduced later in the term. Feel free to experiment with them beforehand if you wish.