Teaching Supply Chain Concepts with the Newsboy Model

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Abstract

This paper presents an alternative method of teaching the standard Newsboy model. In additon to a retailer, who orders using the Newsboy quantity, we also examine the profit of a manufacturer who supplies the retailer. We provide the reader with several solved homework problems that can generate significant student interest. The first addresses the effect of different wholesale prices and illustrates double marginalization. The second applies the model to a problem of postponing differentiation, the third deals with market structure, and the fourth involves a supply contracting problem using the theory of real options. We found that this method of teaching, and the supply chain exercises, generated significant student interest in an MBA elective.

Keywords: Inventory Models, Newsboy Model, Real Options, Supply Chain Management

The Newsboy model is a *single period* inventory model often used for short life-cycle products like fashion apparel, shoes, sporting goods, furniture and toys. Certain popular case studies, such as Sport Obermeyer (Hammond and Raman, 1994) and Mattel: Vendor Operations in Asia (Johnson, 1998), address the supply chain issues of short life cycle products. Many instructors use these cases in conjunction with the Newsboy model to provide students with both strategic concepts and tactical tools (Johnson and Pyke, 2000). Researchers have also employed the Newsboy model to investigate supply chain problems (Rudi and Zheng, 1997; Corbett and Tang, 1999, for example). This model has been taught in Inventory electives, and more recently, in core Operations Management courses at both business schools and in industrial engineering programs. Most textbooks do not provide a formal derivation of the solution because of the assumed lack of capability of the students, or they apply Leibnitz's rule and place it in an appendix. See Krajewski and Ritzman (1999), Nahmias (1997) and Silver, Pyke and Peterson (1998) for instance.

In this paper, we present an alternative method of teaching the standard Newsboy model that does not rely on Leibnitz's rule and therefore is much more intuitive and accessible for many students. We begin by developing a technical note that can be used directly by students. It is contained in Sections 1 and 2 of this paper and is intended as stand-alone materials. In the first part of the note – Section 1 of this paper – we develop an intuitive expression for the expected profit function, transforming it to a form in which underage and overage cots are isolated. This form allows students to understand important managerial insights. We derive the retailer's optimal order quantity and discuss its impact on the retailer's and manufacturer's profit. We demonstrate how these values can be computed in Excel. Finally, we demonstrate interesting applications of the Newsboy model using solved problems. These illustrate concepts of double marginalization, postponement and different market structure. Some of the problems are intended as student homework exercises, so answers are provided in the Appendix.

In the second section, we introduce a supply contracting problem that utilizes some concepts from real options theory as a method to share risk between the retailer and the manufacturer. The results are identical to Pasternack (1985), but we found the concept of options to have higher appeal to finance-oriented MBA students than return prices. We show how the choice of the option price and the exercise price can replicate the optimal solution of an integrated firm as well as creating any allocation of the system profit.

Finally, in the Appendix, we provide a teaching note for conducting a class using the technical note. The teaching note expands on the solutions to the problems and provides insights from our experience with the material in a supply chain elective at Tuck.

Porteus (1990) presents the Newsboy model and several extensions for a more research-oriented audience. The approach presented here was used in Rudi and Zheng (1997) and in Rudi, Kapur and Pyke (1999). Lau (1997) presents formulas for computing the expected cost for several probability distributions. Rudi and Zheng (1997) characterize the optimality conditions of the postponement problem addressed below in Homework Problem #2, and Zemel (1998) has developed a spreadsheet for computing the corresponding expected profit. Finally, see Pasternack (1985), Cachon (1999), Lariviere (1999), Tsay, Nahmias and Agrawal (1999), Corbett and Tang (1999), and Anupindi and Bassok (1999) for other types of supply contracts. The next two sections can be handed out as a reading for students to use in preparation for class discussion.

1 The Newsboy Model

The Newsboy problem derives its name from a common problem faced by a person selling newspapers on the street. The merchant must decide, before observing demand, how many papers to buy, and once the day is over, the newspapers have little value. These decisions appear in a wide array of industries and they are notoriously difficult. Every Christmas, for instance, news reports focus on the inability to find certain extremely popular toys. At the same time, retailers are left with piles of toys that turned out to be less popular than expected.

In general, the Newsboy problem has the following characteristics. Prior to the season, the buyer must decide how many units of each item to purchase. The procurement lead time tends to be quite long relative to the selling season, so the buyer cannot observe demand prior to placing the order. Due to the long lead time, there is no opportunity to replenish inventory once the season has begun. Unfortunately, demand for these items tends to be highly uncertain, often because they are fashion goods whose styles change from season to season. Demand occurs during the selling season. If supply exceeds demand, the excess must be sold at a loss (salvaged), and if demand exceeds supply, the unmet demand is lost. The costs associated with these events are called overage and underage costs. Overage costs capture markdowns or inventory holding costs, if it is possible to carry stock over to the next year. Underage costs include lost profit margin, and perhaps the costs of expediting, buying stock from a competitor to meet the demand, and customer ill will.

1.1 Expected profit

Consider a retailer who sells a short life cycle, or single-period, product. The retailer orders Q units (the decision variable) before the season at unit wholesale price W. Then demand D is observed, and the retailer sells units (limited by the supply Q and the demand D) at unit revenue R (where $R > W$). Any excess units can be salvaged at unit salvage value S. For example, excess units can be sold on sale at a reduced price where $S < W$; they are sold at a loss. Prior to the selling season, demand is uncertain. However, the retailer knows the probability distribution of demand. Most of our analysis allows for a general, continuous demand distribution, but in computations we employ the normal distribution, which is commonly used in practice.

To facilitate the modelling, we will introduce an important definition:

$$y^+ = \begin{cases} y & \text{if } y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

We often call y^+ *the positive part of* y. For example, $3.4^+ = 3.4$ and $(-2)^+ = 0$.

Let us examine each term of the retailer's profit function. The retailer earns revenue from the units sold, and she can recover some of the value of excess units through sale at the salvage value, but she must acquire each unit at the unit acquisition cost.

The number of units sold is limited by the demand D and by the supply Q. If $Q < D$, the retailer can sell Q units, and if $Q \geq D$ she can sell D units. So the revenues obtained by ordinary sales are the unit revenue R times the number of units sold, and can be expressed as:

$$R \min(D, Q)$$

If $Q > D$ then there are $Q - D$ units left over; and if Q is not larger than D, then there are no leftovers. Therefore, the number of units left over can be written as $(Q - D)^+$. The salvage value of the extra units is given by the unit salvage value times the number of units left over, yielding the following expression:

$$S(Q - D)^+$$

Finally, the acquisition cost is simply the unit wholesale price W times the number of units bought, Q. Because it is a cost, it is negative in the profit expression:

$$-WQ$$

Because demand is unknown at the time of decision (i.e. when Q is determined), we are interested in the *expected profit as a function of* Q, which we denote by $\Pi_r(Q)$. From the previous discussion, we have that:

$$\Pi_r(Q) = E\left[R\min(D,Q) + S(Q-D)^+ - WQ\right] \tag{1}$$

1.2 From profit to opportunity cost

We will now manipulate $\Pi(Q)$ to rewrite it in a different form. For the first term, we use the rule:

$$\textit{Rule } 1: \qquad \min(D,Q) = D - (D-Q)^+ \tag{2}$$

Let us verify this equation. If $Q < D$, then $D - (D-Q)^+ = D - (D-Q) = Q$ which is the same as the minimum of the two when Q is smaller than D. If $Q \geq D$, then $D - (D-Q)^+ = D - 0 = D$, which again reproduces the minimum of the two for the case that Q is greater or equal to D. We can conclude that the rule is correct.

We do not rewrite the term reflecting the salvage value, but for the acquisition cost term, we use the following rule:

$$\textit{Rule } 2: \qquad Q = D - (D-Q)^+ + (Q-D)^+ \tag{3}$$

Let us also verify this equation. If $Q < D$ then $D - (D-Q)^+ + (Q-D)^+ =$

$D - (D - Q) + 0 = Q$, which reproduces Q. If $Q \geq D$ we get $D - (D - Q)^+ + (Q - D)^+ = D - 0 + (Q - D) = Q$. Therefore we see that this rule also is correct.

Using these two rules, we can rewrite $\Pi_r(Q)$ as the following expression:

$$\Pi_r(Q) = E\left[R\left(D - (D-Q)^+\right) + S(Q-D)^+ - W\left(D - (D-Q)^+ + (Q-D)^+\right)\right] \tag{4}$$

By collecting similar terms, we get:

$$\Pi_r(Q) = (R - W)ED - \overbrace{E\left[(R-W)(D-Q)^+ + (W-S)(Q-D)^+\right]}^{G_r(Q)} \tag{5}$$

The first term of this expression for the expected profit, $(R-W)ED$, is the *unit margin* times the expected demand (i.e. the forecast). This term is the *expected profit with no demand uncertainty (i.e. perfect information).* The rest of the expression, defined as $G_r(Q)$, is then *the cost of demand uncertainty.* The first term of $G_r(Q)$ is the expected number of units short, $(D - Q)^+$, times the unit opportunity cost per unit short, $R-W$ (the lost margin). This unit opportunity cost is the *unit underage cost*:

$$C_u = R - W$$

Similarly, the second term of $G_r(Q)$ consists of the expected number of leftover units $(Q-D)^+$ times the unit opportunity cost per unit left over which is $W-S$. This unit opportunity cost is the *unit overage cost*:

$$C_o = W - S$$

The expected cost of demand uncertainty can then be rewritten as:

$$G_r(Q) = E\left[C_u(D-Q)^+ + C_o(Q-D)^+\right] \tag{6}$$

1.3 Optimal order quantity

The retailer must determine *the order quantity that will give the highest expected profit.* If we examine expression (5), we see that the first term does not have a Q in it. Therefore, the first term is not affected by the order quantity. Because of this, setting Q to maximize $\Pi_r(Q)$ gives the same result as setting Q to minimize $G_r(Q)$. This minimization is performed by taking the derivative of $G_r(Q)$ and then finding the value of Q that makes the derivative equal to zero.

By looking at expression (6), we see that the change in $(D-Q)^+$ when we increase Q by one unit (i.e. the derivative) is -1 with the probability that $D > Q$. That is, if D is not greater than Q, an increase in Q of one unit has no impact on $(D-Q)^+$ because the expression equals zero. If $D > Q$, an increase in Q by one unit causes $(D-Q)^+$ to decrease by one unit. It follows that the derivative of the first term of (6) is:

$$-C_u \Pr(D > Q)$$

By the same reasoning, the change in $(Q-D)^+$ when we increase Q by one unit is 1 with the probability $\Pr(Q > D)$. Thus, the derivative of the second term of (6) is:

$$C_o \Pr(Q > D)$$

The derivative of $G(Q)$ is then:

$$G_r'(Q) = -C_u \Pr(D > Q) + C_o \Pr(Q > D) \tag{7}$$

Since $\Pr(D > Q) = 1 - \Pr(D < Q)$, we can rewrite the derivative as:

$$G_r'(Q) = -C_u \left[1 - \Pr(D < Q)\right] + C_o \Pr(D < Q) \tag{8}$$

By equating this derivative to zero, and rearranging terms, we get the following *Newsboy optimality condition*:

$$\Pr(D < Q) = \frac{C_u}{C_u + C_o} \tag{9}$$

If we can find the value of Q that satisfies expression (9), then this is the optimal value of Q, which we denote by Q^*.

Normal distribution

In this subsection assume that demand is normally distributed with expected value μ and standard deviation σ. We can then easily find the optimal order quantity using Microsoft's Excel spreadsheet:

$$Q^* = NORMINV(C_u/(C_u + C_o), \mu, \sigma)$$

Example

Let $R = 100$, $W = 50$ and $S = 20$. Then

$$C_u = 100 - 50 = 50$$

and

$$C_o = 50 - 20 = 30$$

The right hand side of the optimality condition then becomes:

$$\frac{50}{50+30} = 0.625$$

Let the demand be normally distributed with mean $\mu = 1000$ and standard deviation $\sigma = 300$. Then the optimal order quantity can be found by entering the following expression in a cell in Excel:

$$= NORMINV(0.625, 1000, 300)$$

The result is $Q^* = 1096$. This means that the optimal decision is to order 96 units more than the forecast.

1.4 The opportunity cost and expected profit

The opportunity cost expression is more difficult to calculate. For the normal distribution, we have the following formula in Excel:

$$G_r(Q^*) = \sigma * (C_u + C_o) * NORMDIST(NORMSINV(C_u/(C_u+C_o)), 0, 1, FALSE) \tag{10}$$

In our example, we can calculate $G(Q^*)$ by writing the following statement in an Excel cell:

$$= 300 * 80 * NORMDIST(NORMSINV(0.625), 0, 1, FALSE),$$

which gives the value $G(Q^*) = 9101$.

Using this result, the expected profit is

$$\Pi_r(Q^*) = 50 * 1000 - 9101 = \$40,899$$

1.5 How about the manufacturer?

Thus far, we have considered only the retailer's decision. Supply chain management, however, is primarily concerned with improving the efficiency of multiple supply chain partners. Therefore, we extend the discussion to include a manufacturer who sells to the retailer. In our modeling scenario, the manufacturer responds to retailer requests by making units to order at unit production cost M. The manufacturer's profit when the retailer orders optimally is then:

$$\Pi_m = (W - M)Q^* \tag{11}$$

Note that there is no uncertainty in this expression since the manufacturer produces to order and takes no risk.

For the normal distribution, the manufacturer's profit can easily be computed in Excel by the following formula:

$$\Pi_m = (W - M) * NORMINV(C_u/(C_u + C_o), \mu, \sigma)$$

If the manufacturer's unit production cost is $M = 30$, we can compute this in Excel by writing the following expression in a cell:

$$= 20 * NORMINV(0.625, 1000, 300)$$

Thus the manufacturer achieves profit of $\Pi_m = \$21,912$.

Now test your understanding by completing the following three sample problems.

1.6 Homework Problem #1: Wholesale price

For the previous example, let the wholesale price W range from 31 to 99. Plot Π_r, Π_m and $\Pi_r + \Pi_m$. Explain the behavior of the three graphs.

1.7 Homework Problem #2: Postponement

Yeah Inc. is a Canadian wholesaler of brightly-colored T-shirts. Yeah orders the T-shirts from a contractor in Bangladesh. The lead time is 6 months, and the selling season is 4 months during the summer. The unit revenue is $R = 2$ while the unit wholesale price of all colors is $W = 1$. The T-shirts come in 4 different colors, with specific parameters given in the following table:

Color	orange	yellow	lime green	signal red
μ	800	300	600	400
σ	300	170	200	130
S	0.75	0.80	0.70	0.75

1. What are the optimal order quantities of the different colors? What is the resulting expected profit? Use the "Postponement" spreadsheet to calculate the expected profit of Yeah Inc. when ordering these quantities.

   This spreadsheet calculates the expected profit, using Monte Carlo integration, for any order quantities of each color. It can be found at

http://mba.tuck.dartmouth.edu/pages/faculty/dave.pyke/papers/Postponement.xls

2. A local dying company, Pink Ink Inc., contacts Yeah and suggests that they *postpone* the application of the color. This change involves buying uncolored (known as *greige*) T-shirts in Bangladesh. Pink Ink offers to color these T-shirts with a lead time of 3 days, implying that they can be colored to order. Using this method, Yeah will take the risk on the total quantity rather of the quantity of each color. The expected demand for the total number of T-shirts is then 2100 with standard deviation $\sqrt{(300^2 + 170^2 + 200^2 + 130^2)} = 419$. Unfortunately for Yeah, the unit cost will increase by 10% with Pink Ink. The unit salvage value of uncolored units is 0.80 (achieved by dying all units yellow and selling at yellow's salvage value). Using the postponement strategy, how many uncolored T-shirts should Yeah order? Using the spreadsheet, what is the expected profit?

3. What is your recommendation to Yeah?

1.8 Homework Problem #3: Evaluating different market structures

NoNo is a producer of high-end YoYo's (they only make one product) at unit production cost \$10. The YoYo's only sell during the summer, and a new model is launched every year. The total production lead time is 6 months.

1. The YoYo is currently sold in 100 different identical retailers. They pay a unit wholesale price of \$15 and sell at unit revenue \$30. Unit salvage value is \$8. Each retailer faces a normal distribution of demand with mean of 100 and standard deviation of 40.

   a. How many YoYo's should each retailer order?

   b. What is the expected profit for each retailer?

   c. What is the manufacturer profit?

2. Mississippi.com - a large Internet retailer - is planning to move into YoYo's. They would pay a unit price of \$13 and sell at unit revenue \$25 (with salvage value the same at \$8). They would expect to get a demand of 5000 units with standard deviation 290.

   The number of retailers would then be reduced to 50, each with expected demand of 120 units and standard deviation of 45. All cost and revenue parameters would stay the same for the retailers.

   a. How many YoYo's should each retailer order?

b. How many YoYo's should Mississippi.com order?

c. What is the manufacturer profit?

3. Rather than using Mississippi.com, the manufacturer can choose to start up an Internet retailer called Nile.com to sell their YoYo's. Nile.com would then sell at the same unit revenue and have the same expected demand and standard deviation as Mississippi.com. There will, however, be some fixed costs associated with running Nile.com.

   What is the maximum fixed cost of Nile.com that makes Nile.com preferable to using Mississippi.com (for the manufacturer)?

2 Supply contract: real option

We now move to a type of supply contract that uses the concept of real options. With this type of contract, the manufacturer offers the following deal to the retailer: Ahead of the season, the retailer buys q call options at unit cost c. Each call option gives the retailer the right to buy a unit of the product at unit exercise price x after the retailer observes the demand. The idea is for the manufacturer to induce the retailer to buy more units, and thus capture more sales, by sharing some of the risk of demand uncertainty with the retailer.

2.1 Expected profit/opportunity cost

Since the retailer needs to exercise one option for every unit sold, the retailer's unit revenue will be adjusted down by the exercise price. The expected profit of the retailer will then be the revenue minus the acquisition cost of the options, given by the following expression:

$$\pi_r(q) = (R - x)\min(D, q) - cq \tag{12}$$

Using our rules of manipulation from the standard Newsboy model, we can rewrite this expected profit as:

$$\pi_r(q) = (R - x - c)ED - \overbrace{E\left[(R - x - c)(D - q)^+ + c(q - D)^+\right]}^{g_r(q)} \tag{13}$$

Let $k_u = R - x - c$ (the unit opportunity cost of buying too few options) and $k_o = c$ (the unit opportunity cost of buying too many options). We then write the expected opportunity cost as:

$$g_r(q) = E\left[k_u(D - q)^+ + k_o(q - D)^+\right] \tag{14}$$

2.2 Optimal number of options

This equation clearly corresponds to the Newsboy expected opportunity cost, and hence has the optimality condition:

$$\Pr(D < q) = \frac{k_u}{k_u + k_o} \tag{15}$$

This can be solved using the corresponding method for the Newsboy model to give the optimal number of options, q^*, to buy.

2.3 Calculating expected profit

The difficult part of the expected profit, namely the expected opportunity cost, can be found for the normal distribution by the Excel formula corresponding to the Newsboy model (for the case in which the retailer buys the optimal number of options):

$$g_r(q^*) = \sigma * (k_u + k_o) * NORMDIST(NORMSINV(k_u/(k_u + k_o)), 0, 1, FALSE) \tag{16}$$

Corresponding to the example for the Newsboy model, the expected profit can be computed by:

$$\pi_r(q^*) = (R - x - c)\mu - g_r(q^*) \tag{17}$$

2.4 Manufacturer's profit

The profit for using real options differs quite a bit from the Newsboy model for the manufacturer. In this case, the manufacturer also faces uncertainty because she does not know how many options will be exercised. Also, it is the manufacturer who is responsible for any extra units and hence will salvage these at the unit salvage value. The manufacturer's expected profit then is:

$$\pi_m(q) = E\left[(c - M)q + x\min(D, q) + S(q - D)^+\right] \tag{18}$$

The first term of this expression is the net revenue received from producing q units at unit cost M and receiving c per unit from the retailer. The second term is the revenue, x, per exercised option, and the third term is the salvage value, S per unit for each option that is not exercised.

For the normal distribution, we have:

$$\pi_m = (c+x-M)\mu - \sigma * ((x-S) * NORMDIST(NORMSINV(k_u/(k_u+k_o)), 0, 1, FALSE)) \quad (19)$$

2.5 Homework Problem #4: Real options

For this problem, use the data from the Newsboy example in Section 1.3 ($R = 100$, $W = 50$, $S = 20$, $M = 30$, $\mu = 1000$ and $\sigma = 300$). Assume the manufacturer is considering two alternative real option contracts with $c = 31$ and $x = 20$, or $c = 6.5$ and $x = 48$.

1. For each of the two contracts, answer the following questions: How many options should the retailer buy? What will be the retailer's expected profit? What will be the manufacturer's expected profit?

2. If the manufacturer and the retailer merge into one company that strives to maximize its total profit, how much should this company produce? What is the resulting expected profit?

3. Compare the results for the Newsboy model (the example used in the text) and the results in questions 1 and 2. What are the reasons for the differences?

Appendix: Teaching note

In teaching this material to second year MBA students in a Supply Chain Management elective, we required students to read a technical note consisting of Sections 1 and 2 of this paper. The postponement and option problems were assigned for homework after the lecture on this material. We discussed solutions and extensions in the following class. Therefore, the students were able to read the technical note, participate in a lecture/discussion on that material, and then exercise their skills using the problems. In this section, we provide a few additional comments on teaching this material based on our experience.

We began the lecture with a general, qualitative discussion of the characteristics of style goods. Students are asked to develop this list, and they mention such characteristics as high markups, sharp discounting, long lead times relative to the product life cycle, and high uncertainty about demand. They are then asked to suggest products that fit into the style goods category (see the list below). The point of this discussion is to convince them that the Newsboy material is important and potentially very useful.

We then present the model following the basic structure of Sections 1 and 2, taking questions and proceeding as slowly as necessary. Our experience is that students follow this material fairly easily, but it is necessary to take plenty of time presenting it. After a few examples, and a brief introduction to the homework, the class ends.

Other examples of the Newsboy problem

Early in the lecture, the class creates a list of examples of products or industries that face the Newsboy problem. For the instructor's convenience we list a few here. (This list is based on Chapter 10 of Silver, et al. (1998).)

1. The garment manufacturer (or retailer): What quantity of a particular style good should be produced (or bought for sale) prior to the short selling season?

2. The Christmas tree vendor: How many trees should be purchased to put on sale?

3. The cafeteria manager: How many hot meals of a particular type should be prepared prior to the arrival of customers?

4. The supermarket manager: How much meat or fresh produce should be purchased for a particular day of the week?

5. The administrator of a regional blood bank: How many donations of blood should be sought and how should they be distributed among hospitals?

6. The supplies manager in a remote region (for example, medical and welfare work in northern Canada): What quantity of supplies should be brought in by boat prior to the long winter freeze-up?

7. The farmer: What quantity of a particular crop should be planted in a specific season?

8. The toy manufacturer: A particular product shows significant potential sales as a "fad" item. How many units should be produced on the one major production run to be made?

The wholesale price problem

This problem is useful for illustrating double marginalization. Answers to the problems can be found in the spreadsheet at

http://mba.tuck.dartmouth.edu/pages/faculty/dave.pyke/papers/Problems.xls

We see from the solution that Π_r is decreasing in W, Π_m looks concave in W, and $\Pi_r + \Pi_m$ is decreasing in W. Also note that the only value for W that will replicate the optimal profit of an integrated firm is $W = R$.

One insight that can be useful to draw in class is that a franchise setup where the retailer pays a fixed membership fee while getting lower unit wholesale price W will create a bigger pie to share between the manufacturer and the retailer.

The postponement problem

This problem illustrates the postponement problem. Out of two *pure* strategies, it shows that full postponement is not always the best approach. The third question will illustrate that it is not necessarily *all or nothing*.

1. The optimal order quantities are $(1052, 464, 747, 509)$ with corresponding expected profit of $\$1,819$. These are found using independent order quan-

tities from the standard Newsboy problem. However, to compare with part 3, one should use the "Postponement" spreadsheet which employs a specific set of random numbers. The expected profit in this case is $\$1,828$.

2. The optimal order quantity of the uncolored option is $2,383$ with corresponding expected profit $\$1,730$. Again, this is found using standard analytical Newsboy formulas. Using the Postponement spreadsheet yields a profit of $\$1,738$.

3. The smart student will identify the opportunity to combine the two technologies. A close-to-optimal solution, found by using the Solver on the Postponement spreadsheet, is $(875, 270, 557, 357)$ for the colored items, and 503 for the greige item, with corresponding expected profit of \$1899. Many other solutions, with expected profit values very close to this, are possible. This might seem like a small increase. But when one considers the fact that the industry bottom line profit averages 3% of revenues, this will have a huge impact on bottom line profit (more than a 50% increase). The optimal solution to this combined strategy is characterized in Rudi and Zheng (1997).

The market structures problem

We found this problem useful in showing that the Newsboy model can be used to quantify the effects of different strategies.

For part 1, we follow the simple Newsboy presentation of Section 1.3. We have $R = 30$, $W = 15$ and $S = 8$. Then

$$C_u = 30 - 15 = 15$$

and

$$C_o = 15 - 8 = 7$$

The right hand side of the optimality condition then becomes:

$$\frac{15}{15 + 7} = 0.682$$

The demand is normally distributed with mean $\mu = 100$ and standard deviation $\sigma = 40$. Then the optimal order quantity can be found by entering the following expression in a cell in Excel:

$$= NORMINV(0.682, 100, 40)$$

The result is $Q^* = 119$.

We can calculate $g(Q^*)$ by writing the following statement in an Excel cell:

$$= 40 * 22 * NORMDIST(NORMSINV(0.682), 0, 1, FALSE)$$

which gives the value $G(Q^*) = 314$

Therefore, the expected profit is

$$\Pi_r(Q^*) = 15 * 100 - 314 = \$1,186$$

The manufacturer's profit, for $M = 10$, is:

$$= 100 * 5 * NORMINV(0.682, 100, 40)$$

Thus the manufacturer achieves profit of $\Pi_m = \$59,456$.

For part 2, we now have $R = 25$, $W = 13$ and $S = 8$ for Mississippi.com. With mean demand of $\mu = 5000$ and standard deviation $\sigma = 290$, Mississippi should order $Q^* = 5157$ YoYo's. The retailers have $\mu = 120$ and $\sigma = 45$, and the cost parameters as before. Therefore, each retailer should order $Q^* = 141$ units.

The manufacturer's profit, for $M = 10$, is based on their sales to Mississippi.com and to the 50 retailers. From the retailers, the manufacturer receives profit of \$35,319, using the previous formula, and from Mississippi, the manufacturer receives \$ 15,471, for a total of \$50,790.

For part 3, Nile.com would have $R = 25$, $M = 10$ and $S = 8$. With mean demand of $\mu = 5000$ and standard deviation $\sigma = 290$, Nile should order $Q^* = 5344$. The retailer results would be identical to those in part 2.

The manufacturer's profit on its own sales would be \$74,027. From the retailers, the manufacturer would earn profit of \$35,319, as in part 2. Thus, the total profit would be \$109,356, or a gain of \$58,556 over the \$50,790 earned from Mississippi. Therefore, the manufacturer could afford a fixed cost of \$58,556 to run Nile.com.

The real options problem

The first real options contract ($c = 31$ and $x = 20$) gives $q = 1086$ with $\pi_r = \$39,809$ and $\pi_m = \$21,000$ which, compared to the standard Newsboy model, is worse for both. (Note the correspondence between the model of real options and the returns problem (Pasternack, 1985). The unit wholesale price in the returns problem is then $c + x$ and the unit payback for returns is x.)

The second contract, however, gives $q = 1345$ with $\pi_r = \$42,289$ and $\pi_m = \$22,771$, which replicates the solution of the integrated firm.

It follows that a real option contract can be very valuable if the parameters are set correctly. If time and student background permits, it may be interesting to look at combinations of c and x that will replicate the expected total profit of an integrated firm. Since the expected profit of an integrated firm is concave in the order quantity, the number of options need to be equal to the optimal order quantity of an integrated firm. To achieve this, we equate the right hand side of the respective newsboy fractiles:

$$\frac{R - x - c}{R - x} = \frac{R - M}{R - S}$$

Solving with respect to x gives

$$x = R - c\frac{R - S}{M - S}, \text{ for } c \in \left(0, R\frac{M - S}{R - S}\right)$$

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