Impact of Supply Learning
When Suppliers Are Unreliable

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Dual sourcing and inventory are two prevalent and widely studied strategies firms use to manage yield risk. A pervasive but implicit assumption in the literature is that a firm knows its suppliers’ yield distributions with certainty. This is a strong assumption in many circumstances. A firm is more likely to have a forecast of a supplier’s yield distribution and to update that forecast based on its experiences with the supplier. We introduce and analyze a Bayesian model of “supply learning” (i.e., distribution updating) and investigate how supply learning influences both sourcing and inventory strategies in dual-sourcing and single-sourcing models, respectively. In the case of Bernoulli all-or-nothing yield distributions, we completely characterize the firm’s optimal sourcing and inventory decisions for the supply-learning model. Among other results, we prove that for a given expected supplier reliability (i.e., the mean of the firm’s forecast for the probability of successful delivery) an increase in the reliability forecast uncertainty increases the attractiveness of a supplier, but it reduces the firm’s desire to invest in inventory to protect against future supply failures. We extend our analysis to allow for general yield distributions, multiple sourcing (i.e., more than two suppliers), and inventory carryover in the dual-sourcing model.

Key words: supply uncertainty; dual sourcing; inventory

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1. Introduction
Supply uncertainty is a challenge faced by numerous firms. One common manifestation of supply uncertainty is random yield, in which the firm receives a random portion of the order placed with a supplier. Two prevalent strategies for managing yield risk are dual sourcing and inventory. Dual sourcing, whereby the firm places orders with two suppliers, mitigates the current period’s supply risk because the probability of both suppliers having low yields is smaller than that of a single supplier having a low yield. Inventory investment, whereby the firm orders more than its current-period needs, mitigates future supply risk because, assuming a high yield, the firm will have additional units on hand in case of low yields in the future.

To the best of our knowledge, the existing supply-uncertainty literature has assumed that firms know their suppliers’ true supply distributions. The reality, however, is that firms often do not know the true distributions and, therefore, must make sourcing and inventory decisions based on their beliefs regarding supply uncertainties. Firms can, however, learn about supply distributions based on their experience with suppliers, and what they learn should influence future sourcing and inventory decisions. In this paper we introduce and analyze a model of supply learning and investigate how supply learning influences both sourcing and inventory strategies.

In many cases, supply uncertainty is binary in nature, that is, an order placed with a supplier either arrives in full or not at all. Such all-or-nothing supply processes arise in practice for various reasons. Natural disasters, equipment failures, fires, strikes, and terrorism can lead to supply disruptions in which a supplier is temporarily down. Batch failures, in which the whole order is lost, are quite common in biomanufacturing (Fox et al. 2003). Acceptance sampling leads to a binary supply process because deliveries are either accepted or rejected in total, even if the actual yield of good units in the delivery is not all-or-nothing. Nahmias (2005, p. 645) notes that “acceptance sampling will remain a valuable tool for many years to come.” This is especially true for industries, such as
the pharmaceutical industry, that do not have highly capable processes. According to a Biogen executive, “on average, the pharmaceutical industry operates at one-sigma quality levels” (DePalma 2003, p. 37).

We focus on all-or-nothing yield uncertainty in this paper but also consider general yield distributions. We use the term reliability to refer to the probability of a successful delivery in the all-or-nothing case. The central question that we investigate is how uncertainty about a supplier’s true reliability influences a firm’s optimal sourcing and inventory decisions. Does reliability uncertainty make a supplier more or less attractive? Does it increase or decrease the firm’s incentive to invest in inventory? Using a Bayesian approach for supply learning (i.e., reliability-forecast updating), we characterize the firm’s optimal sourcing and inventory decisions. Among other results, we prove that for a given expected supplier reliability (i.e., the mean of the firm’s forecast for a supplier’s reliability), increasing forecast uncertainty (as measured by the squared coefficient of variation) increases the attractiveness of a supplier, but it reduces the firm’s desire to invest in inventory to protect against future supply failures.

The remainder of this paper is organized as follows. In §2 we review the relevant literature. The model is described in §3. We investigate the impact of supply learning on the firm’s sourcing strategy and inventory investment in §§4 and 5, respectively. Extensions are presented in §6. Conclusions are presented in §7. Appendices are contained in an online technical supplement that accompanies this paper. All proofs are contained in Appendix 3.

2. Literature Review

The random-yield literature has a rich history dating back to Karlin (1958a, b). We concentrate on the papers most directly relevant to our research and refer the reader to the survey paper of Yano and Lee (1995) for a comprehensive review of the literature.

Much of the random-yield literature focuses on developing the optimal inventory policy for single-supplier models. Gerchak et al. (1988) study a periodic-review, finite-horizon problem with stationary, stochastic demand and stationary, stochastically proportional random yield. Order costs are linear in the realized yield. The authors prove that the optimal ordering policy is not a simple order-up-to type. Henig and Gerchak (1990) extend this work to allow for more general cost structures in which the procurement cost can depend on both the order size and the quantity received. While the vast majority of the random-yield literature assumes a stationary yield, Parlar et al. (1995) allow for nonstationary yield by assuming the yield follows a Markov modulated Bernoulli (MMB) process; an order either succeeds or fails and the probability of success evolves in a Markovian fashion. The order cost has a fixed and a variable component. The authors prove that a state-dependent \((s, S)\) policy is optimal. For single-supplier systems that can be modeled by a discrete-time Markov process (e.g., MMB yields), Song and Zipkin (1996) establish that state-dependent \((s, S)\) policies are optimal in general. Song and Zipkin explicitly assume that the “supply system is exogenous, that is the evolution is independent of all other events, including our orders and demands” (p. 1410). We note that an exogenous evolution is also implicitly assumed in the MMB yield model of Parlar et al. (1995). As will be seen later, our supplier-reliability forecasting approach will result in a MMB yield model in which the evolution is endogenous, that is the evolution of the supply system, or more precisely the firm’s forecast of the supply system, does depend on the firm’s ordering decisions. We will also extend our model to allow for general yield distributions.

While the vast majority of the random-yield literature has focused on single-supplier systems in which inventory (through the order-size decision) is used to mitigate future supply uncertainty, a number of papers have explored supplier diversification. Agrawal and Nahmias (1997) investigate the effect of random yields in a single-period, deterministic-demand, supplier-rationalization problem. Dada et al. (2007) consider a newsvendor problem with \(N\) suppliers that have random output functions and they establish a ranking result for supplier selection. Tomlin and Wang (2005) consider the value of dual sourcing and mix flexibility in multiproduct newsvendor networks subject to Bernoulli failures. Babich (2006) and Babich et al. (2007) both consider the impact of competition in two-supplier, Bernoulli yield

In all of the above papers, the ordering firm has perfect knowledge of its supplier yield distributions. A similar assumption also holds true in the supplier-disruption literature (see Tomlin 2006 and references therein); the ordering firm knows its supplier’s failure and repair distributions. In reality, firms do not have perfect information on supply distributions and must make diversification and inventory decisions based on imperfect knowledge that can be refined over time as the firm learns from its experiences with a supplier.

There is a long history, dating back to Dvoretzky et al. (1952) and Scarf (1959), of studying inventory policies in the presence of demand learning. More recent work includes Azoury (1985), Nahmias (1994), and Lariviere and Porteus (1999), among others. There has not been an equivalent focus on inventory or sourcing models in the presence of supply learning. In fact, to the best our knowledge, ours is the first paper to consider the impact of supply learning (or reliability forecasting) on inventory and sourcing decisions, and this is the primary contribution of the paper. We note that in a system-reliability setting, Özekici and Soyer (2003) present a Bayesian approach to forecasting reliability. There are, however, no operational decisions to be made in their setting because the replacement policy is treated as a given: “Whenever the system fails it is replaced by a new and identical one” (p. 131). As in Özekici and Soyer (2003), we propose a Bayesian forecasting approach for a Bernoulli supply process, but in contrast we explicitly consider operational decisions, those being supplier selection and order quantities in our setting. Furthermore, as discussed above, the Markovian evolution depends directly on these operational decisions.

3. Model

We consider a risk-neutral firm that can source from two suppliers \( i = 1, 2 \) over a finite-horizon with \( T \) discrete periods. We index time in a backward manner so that the first period is labeled \( T \) and the last period is labeled 1. In each period \( t \) the firm chooses a quantity \( q_i \geq 0 \) to source from supplier \( i \). We use subscripts to denote the time period and superscripts to denote the supplier. Any exceptions to this convention will be clear from the context. Suppliers often impose minimum-quantity restrictions to cover fixed costs of production. We assume that supplier \( i \) imposes a per-period minimum order quantity \( m_i^* > 0 \); the firm must order at least \( m_i^* \) if it places an order with supplier \( i \). The firm can, however, choose not to place an order with a supplier.

Each supplier is unreliable in the sense that an order placed with supplier \( i \) might succeed or fail, i.e., we assume a Bernoulli yield model. The probability of success is given by \( \theta^i \), and we refer to \( \theta^i \) as the reliability of supplier \( i \). As is common in the literature, we assume that supplier yields are independent and that yields are stationary, that is, the \( \theta^i \) are constant over time. We note that yield stationarity is a reasonable assumption in the pharmaceutical and biomanufacturing industries because “no one wants to spend millions of dollars developing a method to improve yield, then find that the FDA won’t approve it” (Roth 2003). We assume that the firm knows \( \theta^1 \) with certainty. In contrast, the firm has only a forecast of supplier 2’s reliability at the start of the horizon, with the forecast being represented by a probability density function \( h(\theta^2) \). In essence, we assume that the firm has prior experience with supplier 1 but not with supplier 2. We note that while we use the term “supplier,” supplier 1 might be the firm itself for cases where it is starting to use a second, outside source for a component that is manufactured in house.

Much of the supply-uncertainty literature assumes that the cost of an order depends only on the realized quantity and is independent of the order quantity. This reflects reality for some firms but not for all firms. For instance, in the pharmaceutical contract-manufacturing industry, “sometimes batches fail...[but]...those clients who pay on a ‘dollars per batch’ or ‘time and materials’ basis tend to pay, at least partially, for problems that arise. A client who...
pays on a ‘dollar per kg’ basis will not pay for a failed batch because no product was delivered” (Fox et al. 2003). As with Henig and Gerchak (1990) and Tomlin and Wang (2005), we allow for a cost structure that is a function both of the quantity ordered and the quantity delivered. We assume a linear cost structure; if the firm places an order of $q_i$ with supplier $i$, it pays $(c_i + g_i)q_i$ if the order succeeds and $c_iq_i$ if the order fails; $c_i$ is the cost per unit ordered and $g_i$ is the cost per unit delivered. Supplier lead times are assumed to be zero, a common assumption in the random-yield literature; e.g., Gerchak et al. (1988), Henig and Gerchak (1990), Anupindi and Akella (1993), and Swaminathan and Shanthikumar (1999).

Demand in each period is drawn from a stationary, continuous distribution with support on the interval $[x_{\min}, x_{\max}]$ where $0 \leq x_{\min} \leq x_{\max} \leq \infty$. The density function is denoted by $f(x)$ and the cumulative distribution function by $F(x)$. Demand not filled in a period is lost, with a lost sales cost of $\pi$ per unit. Inventory left over at the end of the horizon can be salvaged at a value of $v$ per unit with $v < c_i + g_i$; $v < 0$ implies a disposal cost and $v > 0$ implies a salvage value. If inventory cannot be carried from one period to the next, then left-over inventory at the end of any period can be salvaged at a value of $v$ per unit. If inventory can be carried from one period to the next, then it is charged at $h$ per unit. All costs are discounted at a per-period rate of $0 < \delta \leq 1$, where $\delta = 1$ implies no discounting.

In §3 we will assume inventory cannot be carried from one period to the next, but we will allow inventory to be carried in §4 when we restrict attention to a single-supplier model. In §6, we will extend our model to allow for general yield distributions, a general number of suppliers, and inventory carryover in the two-supplier case.

4. Impact of Reliability Uncertainty on the Sourcing Strategy

In this section, we investigate how supplier-reliability uncertainty influences the sourcing strategy used by the firm to mitigate supply risk. By sourcing strategy we mean the selection of suppliers in any given period, i.e., should the firm dual source or single source; and if it should single source, then which supplier should it use. To focus on the sourcing lever for mitigating supply risk, in this section we assume that the firm cannot carry inventory from one period to the next. While this is a stylized assumption that allows us to isolate the impact of reliability uncertainty on sourcing decisions, we also note that many industries that are subject to significant supply risk, e.g., the biomanufacturing industry, are also subject to inventory perishability.

Recall that the firm can choose whether or not to order from a supplier, but that if it does order from a supplier it must order a quantity greater than or equal to that supplier’s minimum order quantity. For later use, we define the following sets:

\[
Q_m = \{q_i \in \mathbb{R} : q_i \geq m_i\}, \quad i = 1, 2, \\
Q' = Q_m \cup \{0\}, \quad i = 1, 2, \\
Q = \{q \in \mathbb{R}^2 : q_1 \in Q_1, q_2 \in Q_2\}.
\]

That is, $Q'$ is the set of feasible orders from supplier $i$, $i = 1, 2$, and $Q$ is the set of feasible order vectors $q = (q_1, q_2)$. By convention, $q = 0$ is a feasible “order.”

4.1. Sourcing Strategy When Supplier Reliability Is Known

We first develop the optimal policy for the case in which there is no uncertainty about supplier reliability, that is, the firm knows supplier 2’s reliability $\theta^2$. This perfect-information case will serve as a benchmark for investigating the impact of reliability uncertainty on the sourcing strategy. Inventory is perishable, unfilled sales are lost, and all parameters are stationary. The finite-horizon problem can, therefore, be decomposed into $T$ identical single-period problems, so we suppress the time subscript in this subsection. Let $\tilde{C}(q; \theta^1, \theta^2)$ denote the single-period expected cost as a function of the order vector $q = (q_1, q_2)$. Then,

\[
\tilde{C}(q; \theta^1, \theta^2) = (c_1 + \theta^1 g_1)q_1 + (c_2 + \theta^2 g_2)q_2 \\
+ \theta^1 \theta^2 L(q_1 + q_2) + \theta^1 (1 - \theta^2) L(q_1) \\
+ (1 - \theta^1) \theta^2 L(q_2) + (1 - \theta^1)(1 - \theta^2) \pi \mu_x, \tag{2}
\]

where

\[
L(q) = \pi \int_q^{x_{\max}} (x - q) f(x) dx - \int_q^q (q - x) f(x) dx,
\]
and $\mu_x$ is the expected demand. The firm’s problem is to minimize $\tilde{C}(q; \theta^1, \theta^2)$ subject to $q \in Q$. Section A1 (in the technical supplement) contains a complete characterization of the optimal solution to this problem along with some additional theoretical results, and we briefly summarize the results here. There are four possible sourcing strategies: dual source, single source from supplier 1, single source from supplier 2, or zero source, i.e., source nothing from either supplier. Single sourcing from supplier 1 or 2 is optimal for a wide range of reliabilities. Dual sourcing can be optimal for certain reliability pairs, but only if the lost-sale cost is high relative to the procurement costs. Dual sourcing is less likely to be optimal if suppliers differ significantly in their reliabilities. Zero sourcing can be optimal, but only if both supplier reliabilities fall below their respective viability thresholds—a situation that is highly unlikely to arise in practice. Figure 1 illustrates the optimal sourcing strategy and quantities as a function of $(\theta^1, \theta^2)$ for the case of deterministic demand equal to $d$. (Expressions referenced in the figure can be found in §A1.) Specifying the optimal sourcing vector in the stochastic-demand case is somewhat more complicated (see §A1) but the relative positioning of the four basic strategies remains the same.

4.2. Optimal Sourcing Strategy for Reliability Uncertainty

Having characterized the optimal sourcing strategy for the perfect-information case, we are now in a position to investigate how reliability uncertainty influences the firm’s sourcing strategy. To do so, we must characterize the optimal ordering policy when the supplier only has a forecast of a supplier’s reliability. As discussed above, we assume that the firm knows supplier 1’s reliability with certainty, but it has only a forecast of supplier 2’s reliability. The firm’s forecast is represented by a probability density $h(\theta^2)$ for supplier 2’s reliability.

We first consider the single-period case. The firm’s problem is to minimize $E_w[\tilde{C}(q; \theta^1, \theta^2)]$ subject to $q \in Q$, where $\tilde{C}(q; \theta^1, \theta^2)$ is given in (2). $\tilde{C}(q; \theta^1, \theta^2)$ is linear in $\theta^2$ and, therefore,

$$E_w[\tilde{C}(q; \theta^1, \theta^2)] = \tilde{C}(q; \theta^1, \bar{\theta}^2),$$

(3)

where $\bar{\theta}^2$ is the expected value of the firm’s forecast of supplier 2’s reliability. We have a certainty equivalence result; the optimal solution to the single-period, uncertain-reliability problem can be found by solving the single-period, perfect-information problem with supplier 2’s reliability set to the forecast mean. Therefore, uncertainty about supplier 2’s reliability has no impact on the optimal sourcing strategy in the single-period case; the firm’s strategy will be the same if the forecast mean is the same as the actual reliability in the perfect reliability-information case.

We now consider the firm’s sourcing problem when there are $T > 1$ periods in the horizon. In each period $t = T, \ldots, 1$ the firm’s problem is to choose an order-quantity vector $q_t = (q_{1t}, q_{2t})$ that minimizes its total expected discounted cost for the remaining horizon, namely period $t$ through period 1. At the start of the horizon the firm has a forecast of supplier 2’s reliability. Hereafter, we will simply refer to the reliability forecast, with the understanding that it refers to supplier 2’s reliability. We assume that the reliability forecast is a member of the Beta family of distributions, so the density of the forecast is

$$h(\theta^2) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)}(\theta^2)^{\alpha-1}(1 - \theta^2)^{\beta-1},$$

where $\alpha > 0$, $\beta > 0$, and $\Gamma(\cdot)$ is the Gamma function. We note that the expected value of a Beta($\alpha, \beta$) distribution is $\alpha/(\alpha + \beta)$.

The Beta family has a number of properties that make it an attractive choice as a model for reliability
uncertainty. First, its members are all continuous distributions restricted to $0 < \theta < 1$. Second, it is a rich family containing U-shaped distributions, unimodal-symmetric distributions, unimodal-skewed distributions, and the uniform distribution. As such, it can reflect a wide class of initial beliefs. Third, it is a conjugate distribution for a stationary Bernoulli random variable; if the prior distribution is a member of the Beta family, then after a Bernoulli trial the posterior distribution will also be a member of the Beta family. In particular, if the prior distribution is Beta($\alpha$, $\beta$) then the posterior distribution will be Beta($\alpha + 1$, $\beta$) if the trial is successful and Beta($\alpha$, $\beta + 1$) if the trial is unsuccessful. In what follows we will denote the reliability forecast by the ($\alpha$, $\beta$) parameters, with the understanding that the forecast has a Beta distribution. Furthermore, we will not subscript the ($\alpha$, $\beta$) parameters with a time index because the time period will be clear from the context.

The firm obtains information about supplier 2’s reliability if and only if (hereafter, iff) it orders a positive quantity from supplier 2. Let us define the following sets:

$$Q^S = \{q \in \mathbb{R}^2: q^1 \in Q^I, q^2 \in Q^m_m\},$$

$$Q^{N2} = \{q \in \mathbb{R}^2: q^1 \in Q^I, q^2 = 0\},$$

where $Q^I$ and $Q^m_m$ were defined in (1) above. That is, $Q^S$ is the set of feasible order vectors such that supplier 2 is used, and $Q^{N2}$ is the set of feasible order vectors such that supplier 2 is not used. Note that $Q^S \cap Q^{N2} = \emptyset$ and $Q^S \cup Q^{N2} = Q$, the set of all feasible order vectors. Also note that supplier 1 might or might not be used in $Q^S$, and the same holds for $Q^{N2}$. Let the firm’s reliability forecast be ($\alpha$, $\beta$) at the start of period $t$. If the firm orders $q_i \in Q^S$, then its reliability forecast in period $t - 1$ will be ($\alpha + 1$, $\beta$) with probability $\alpha/($$\alpha + \beta$) and ($\alpha$, $\beta + 1$) with probability $\beta/($$\alpha + \beta$). If the firm orders $q_i \in Q^{N2}$, then its reliability forecast in period $t - 1$ will be ($\alpha$, $\beta$) with probability one.

Inventory cannot be carried from one period to the next, and unfilled sales are lost. Therefore, the starting inventory for all periods is zero, so the total expected discounted cost over the remaining horizon (hereafter referred to as the cost-to-go) depends on past events only through the current reliability forecast ($\alpha$, $\beta$). For $t = T, \ldots, 1$, we define the following three cost-to-go values for any state ($\alpha$, $\beta$):

(i) $V_i(\alpha, \beta)$ is the optimal cost-to-go,

(ii) $V^{S2}_i(\alpha, \beta)$ is the optimal cost-to-go subject to ordering from supplier 2 in period $t$, i.e., if an order vector $q_i \in Q^S$ is used, and

(iii) $V^{N2}_i(\alpha, \beta)$ is the optimal cost-to-go subject to not ordering from supplier 2 in period $t$, i.e., if an order vector $q_i \in Q^{N2}$ is chosen. For notational convenience, define $V_0(\alpha, \beta) = 0\forall(\alpha, \beta)$. Then, for $t = T, \ldots, 1$,

$$V_i^{S2}(\alpha, \beta) = \min_{q_i \in Q^S}\{\tilde{C}(q_i; \theta^1, \frac{\alpha}{\alpha + \beta}) + \delta(\frac{\alpha}{\alpha + \beta} V_{i-1}(\alpha + 1, \beta) + \frac{\beta}{\alpha + \beta} V_{i-1}(\alpha, \beta + 1))\},$$

$$V_i^{N2}(\alpha, \beta) = \min_{q_i \in Q^{N2}}\{\tilde{C}(q_i; \theta^1, \frac{\alpha}{\alpha + \beta}) + \delta V_{i-1}(\alpha, \beta)\},$$

$$V_i(\alpha, \beta) = \min\{V^{S2}_i(\alpha, \beta), V^{N2}_i(\alpha, \beta)\},$$

where we have used the certainty equivalence result of (3) and the fact that $\tilde{\theta} = \alpha/($$\alpha + \beta$).

Let $q^*_i(\alpha, \beta)$ denote the optimal order vector in period $t$, i.e., $q^*_i(\alpha, \beta) = \arg\min_{q_i \in Q} V_i(\alpha, \beta)$. Let us define the following two order vectors:

$$q^{S2}(\theta^1, \theta^2) = \arg\min_{q_i \in Q^S}\{\tilde{C}(q_i; \theta^1, \theta^2)\},$$

$$q^{N2}(\theta^1) = \arg\min_{q_i \in Q^{N2}}\{\tilde{C}(q_i; \theta^1, \theta^2)\}.$$
firm chooses to source from supplier 2, then it uses the order vector \( q^{S_2}(\theta^1, \alpha/(\alpha, \beta)) \) that minimizes the expected current period cost. It can ignore the reliability uncertainty because of the certainty equivalence result shown for the single-period model. If the firm chooses not to source from supplier 2, then it simply selects the sourcing quantity vector \( q^{N_2}(\theta^1) \) that minimizes the expected current-period cost. We caution that this theorem does not imply that reliability uncertainty has no impact, only that it has no impact once the firm decides whether or not to source from supplier 2. As we will see, reliability uncertainty does influence whether the firm should source from supplier 2 or not.

Define \( P_t \) as the set of states \((\alpha, \beta)\) for which it is optimal to source from supplier 2 in period \( t \). From Theorem 1, the optimal order vector for \((\alpha, \beta) \in P_t\) is \( q^{S_2}(\theta^1, \alpha/(\alpha, \beta)) \). We note that \((\alpha, \beta) \in P_t\) implies that supplier 2 is used in period \( t \), but it does not necessarily imply single sourcing from supplier 2; dual sourcing can be optimal for many values of \( \alpha/(\alpha + \beta) \). If \((\alpha, \beta) \notin P_t\), then, by definition, it is optimal not to source from supplier 2, i.e., the optimal order is \( q^{N_2}(\theta^1) \).

**Theorem 2.** \( P_{t-1} \subseteq P_t \) for \( t = T, \ldots, 2 \).

The \( P_t \) sets are nested and, therefore, the complement of \( P_t \) is a stopping set. In other words, if it is not optimal to source from supplier 2 in period \( t \), i.e., \((\alpha, \beta) \notin P_t\), then it is not optimal to source from supplier 2 in any remaining period \( t - 1, \ldots, 1 \). If \((\alpha, \beta) \notin P_t\), then single sourcing from supplier 1 for the rest of the horizon is optimal if supplier 1 is viable, i.e., \( \theta^1 \geq \gamma_1^* \) (see §A1) and sourcing nothing for the remainder of the horizon is optimal otherwise. The firm will source from supplier 2 initially iff the initial reliability forecast is contained in \( P_t \), and it will continue to source from supplier 2 iff the (updated) reliability forecast remains in the \( P_t \) sets. We therefore refer to the \( P_t \) as continuation sets.

We now proceed with our development of the optimal ordering policy by characterizing the continuation sets \( P_t \) for \( t = T, \ldots, 1 \). If at the start of period \( t \), the firm chooses not to order from supplier 2 for the remainder of the horizon, then the firm’s problem becomes one of choosing how much to order from supplier 1 in each of the remaining periods. Because the firm has perfect information about supplier 1’s reliability, this finite-horizon problem decomposes into \( t \) identical single-period problems. Therefore, if at the start of period \( t \) the firm chooses to stop ordering from supplier 2, the corresponding expected cost-to-go is given by \( \sum_{\tau=t}^{t} \delta^{t-\tau-1} \tilde{C}_{S_2}(\theta^1) \), where \( \tilde{C}_{S_2}(\theta^1) = \tilde{C}(q^{N_2}; \theta^1, \theta^2) \) is the optimal single-period expected cost subject to not using supplier 2. For later use, let us also define \( \tilde{C}_{S_2}(\theta^1, \theta^2) = \tilde{C}(q^{N_2}; \theta^1, \theta^2) \), i.e., \( \tilde{C}_{S_2}(\theta^1, \theta^2) \) is the optimal single-period expected cost (knowing \( \theta^2 \)) subject to using supplier 2. Define

\[
Z_t(\alpha, \beta) = V_t^{S_2}(\alpha, \beta) - \sum_{\tau=t}^{T} \delta^{t-\tau-1} \tilde{C}_{S_2}(\theta^1),
\]

and \( B_t(\alpha, \beta) \) as the minimum \( \alpha \geq 0 \) such that \( Z_t(\alpha, \beta) \leq 0 \).

**Theorem 3.**

(i) \( Z_t(\alpha, \beta) \leq 0 \Leftrightarrow (\alpha, \beta) \in P_t \).

(ii) \( B_t(\beta) = 0 \) if \( Z_t(0, \beta) \leq 0 \), \( B_t(\beta) = \infty \) if \( Z_t(\infty, \beta) > 0 \), and, otherwise, \( B_t(\beta) \) is the solution to \( Z_t(\alpha, \beta) = 0 \).

(iii) \( B_t(\beta) \) is increasing in \( \beta \).

(iv) \( B_t(\beta) - 1 < B_{t+1}(\beta) \leq B_t(\beta) \) for \( t = T - 1, \ldots, 1 \).

In other words, \( P_t \) is the lower level set \{ \((\alpha, \beta) \in \mathbb{R}_+^2; Z_t(\alpha, \beta) \leq 0 \} \). These continuation sets are defined by increasing boundary functions, \( B_t(\beta) \), that bisect the positive \((\alpha, \beta)\) orthant. The boundary function for the final period, \( B_1(\beta) \), is a straight line emanating from the origin. This reflects the certainty equivalent result for a single-period problem, i.e., only the forecast mean influences the final-period sourcing decision. We note that \( B_1(\beta) \rightarrow \infty \) as \( \beta \rightarrow \infty \), and from (iv) above, so do all \( B_t(\beta) \). While closed form solutions for \( B_t(\beta) \) and \( B_1(\beta) \) can be obtained for the case of deterministic demand, closed-form solutions for \( B_t(\beta) \) do not exist in general. They can, however, be readily calculated in a recursive fashion. The boundary functions are illustrated in Figure 2 for the following parameters \( \delta = 0.99, \pi = 80, v = 4, c^1 = 5, g^1 = 20, \)

\[
m^1 = 25, \theta^1 = 0.9, c^2 = 4, g^2 = 15, m^2 = 25 \text{ and demand being uniformly distributed, } U(80, 120).
\]

Bringing together Theorems 1, 2, and 3, we can now completely characterize the optimal ordering policy.

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1 Throughout the paper we use the terms increasing and decreasing in the weak sense, i.e., nondecreasing and nonincreasing.
the attractiveness of supplier 2 is therefore a viable option during the horizon. One measure of be in the initial continuation set if supplier 2 is to be
functions.  

Theorem 4. In period $t = T, \ldots, 1$,

\[
\alpha < B_t(\beta) \Rightarrow q^*_t(\alpha, \beta) = q^N N_2(\theta^1), \\
\alpha \geq B_t(\beta) \Rightarrow q^*_t(\alpha, \beta) = q^2 (\beta^1, \frac{\alpha}{\alpha + \beta}).
\]

Supplier 2 is used by the firm iff the initial forecast parameters satisfy $\alpha \geq B_t(\beta)$, i.e., the initial state must be in the initial continuation set if supplier 2 is to be a viable option during the horizon. One measure of the attractiveness of supplier 2 is therefore $B_t(\beta)$; as $B_t(\beta)$ decreases the continuation set enlarges, so supplier 2 is used in more states. The following theorem establishes the comparative statics for the boundary functions.

Theorem 5. For $t = T, \ldots, 1$, $B_t(\beta)$ is decreasing in supplier 1 cost components $c^1$ and $g^1$, and in supplier 1’s minimum order quantity $m^1$, but is increasing in supplier 1’s reliability $\theta^1$ and in supplier 2 parameters $c^2$, $g^2$, and $m^2$.

This result highlights the tactics that a supplier can use to win business from a firm who is uncertain about that supplier’s reliability. For example, decreasing the minimum order quantity makes supplier 2 more attractive, one reason being that it reduces the reliability exploration cost for the firm. Absolute costs are, of course, another lever, but a supplier can also use the relative cost structure to increase its attractiveness. As discussed earlier, cost structures in the contract pharmaceutical-manufacturing industry vary in terms of payment responsibility for failed orders. For a constant total cost of a good unit (i.e., $c^2 + g^2$), one can prove that supplier 2 becomes more attractive as $c^2/g^2$ decreases, i.e., as the firm bear less financial responsibility for failed orders.

4.3. Implications of Reliability Uncertainty for the Sourcing Strategy

Having characterized the optimal sourcing strategy for both the perfect reliability information case and the uncertain reliability case, we are now in a position to consider the implication of reliability uncertainty. To this point we have parameterized the firm’s forecast of supplier 2’s reliability by the parameters $(\alpha, \beta)$. In what follows, we will at times parameterize the forecast by its mean and squared coefficient of variation (hereafter, SCV). We denote the mean and SCV by $\bar{\theta}$ and $\upsilon$, respectively. As with the $(\alpha, \beta)$ parameters, we do not superscript $\bar{\theta}$ and $\upsilon$ with the supplier index “2,” but the reader should understand that $\bar{\theta}$ and $\upsilon$ refer to the firm’s forecast of supplier 2’s reliability. For a Beta distribution, $\bar{\theta} = \alpha/(\alpha + \beta)$ and $\upsilon = \beta/(\alpha(\alpha + \beta + 1))$, and there is a one-to-one mapping between a $(\bar{\theta}, \upsilon)$ pair and an $(\alpha, \beta)$ pair. For a given $\bar{\theta}$, the firm is more certain regarding its forecast as $\upsilon$ decreases, and the perfect information case of §4.1 is obtained as $\upsilon \to 0$.

4.3.1. Value of Perfect Reliability Information.

The firm might be able to take some action other than tracking order realizations to improve its supplier-reliability forecast. For example, it might request that the supplier provide historic yield data for related products. Spending money to improve forecast accuracy is justified only if the benefit of forecast improvement outweighs the associated cost. In this subsection, we consider the benefit of obtaining perfect reliability information. In determining the benefit, we take the so-called frequentist approach to evaluating policies. That is, we determine the expected cost of a policy with respect to supplier 2’s true underlying Bernoulli yield random variable, $\theta^2_{\text{true}}$. This contrasts to a Bayesian approach in which the expectation would be taken with respect to the firm’s subjective probability distribution of supplier 2’s reliability.

We define $V^*_{Tt}$ as the optimal expected cost in a $T$-period problem if $\theta^2_{\text{true}}$ were known with certainty,
i.e., perfect information. We define $V^H_T(\alpha, \beta)$ as the expected cost in a $T$-period problem if the firm’s initial beliefs are given by $(\alpha, \beta)$, i.e., imperfect information, and it uses the optimal Bayesian policy. Both expected costs can be evaluated exactly, but the computation time increases rapidly with the horizon length. We use

$$\Psi^H = \frac{V^H_T(\alpha, \beta) - V^H_T}{V^H_T}$$

as the performance metric; the closer $\Psi^H$ is to zero the better the Bayesian policy performs, so the less value there is to having perfect reliability information.

We present results for two cases. In both cases demand is uniformly distributed between 50 and 150, $\delta = 0.99$, $\pi = 80$, $\nu = 4$, $c^1 = 5$, $g^1 = 20$, $m^1 = 25$, $\theta^1 = 0.9$, $c^2 = 5.25$, $g^2 = 21$, and $m^2 = 25$. In case 1, supplier 2’s true reliability is $\theta^2 = 0.98$. In case 2, supplier 2’s true reliability is $\theta^2 = 0.75$. The initial beliefs are parameterized by the forecast mean and SCV. For each forecast mean $\bar{\theta}$, the SCV was set to half of the maximum possible SCV for a Beta distribution, i.e., $\nu = (1 - \bar{\theta})/(2\bar{\theta})$. Table 1 presents $\Psi^H$ for a range of initial firm beliefs. As can be seen, the Bayesian policy performs very well unless the reliability-forecast mean differs significantly from the actual reliability. This observation held true for other numerical examples tested. This suggests that the performance of the optimal Bayesian policy is robust with respect to the firm’s initial beliefs; therefore, the firm should exercise some caution before investing in forecast improvements.

We also evaluated two other policies: (i) a myopic policy in which the firm determines its sourcing quantities by minimizing the expected current-period cost but the firm does update its beliefs based on the resulting yield realization, and (ii) a policy in which the firm assumes its initial reliability estimate is perfect and never updates this estimate based on yield realizations. Using the same parameters as above (with $T = 15$ and $\theta^2_{\text{True}} = 0.98$), for each of the three policies Figure 3 presents the increase in cost (relative to $V^H_T$, the optimal expected cost knowing the true reliability) as a function of the initial reliability estimate $\bar{\theta}$ for two different minimum-order quantities, $m^1 = 5$ and $m^1 = 50$, $i = 1, 2$. As one might expect, the myopic policy performance is close to that of the optimal Bayesian policy if the initial reliability estimate is reasonably accurate, i.e., close to 0.98, but the performance can be significantly worse if the initial estimate is poor, i.e., when far below 0.98. (However, if one moves far enough below 0.98, then both polices perform identically because supplier 2 is never used in either policy.) In essence, the myopic policy is less robust to the initial beliefs than is the optimal policy. The no-updating policy is even less robust. We also observe that the optimal Bayesian policy is less sensitive to the initial belief for $m^1 = 5$ than for $m^1 = 50$, i.e., the relative cost increase does not increase dramatically until a lower initial reliability estimate is reached for the case of $m^2 = 5$. One reason for this is that “exploration” of supplier 2’s reliability is less costly if the minimum order quantity is lower. In the optimal Bayesian policy, the firm is willing to incur a higher current cost by sourcing from supplier 2.

### Table 1  Value (%) of Perfect Reliability Information as Measured by $\Psi^H$

<table>
<thead>
<tr>
<th>Forecast mean</th>
<th>$\theta_{\text{True}} = 0.98$</th>
<th>$\theta_{\text{True}} = 0.75$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\theta} = a/(a + b)$</td>
<td>$\bar{\theta} = 0.75$</td>
<td>$\bar{\theta} = 0.75$</td>
</tr>
<tr>
<td>$T = 1$</td>
<td>0.6 5.18 5.58 4.67 3.89 0.43 0.43 0.34 0.39</td>
<td>0.6 5.18 5.58 4.67 3.89 0.43 0.43 0.34 0.39</td>
</tr>
<tr>
<td>$T = 5$</td>
<td>0.7 6.54 4.91 3.67 2.97 0.00 0.34 0.51 0.55</td>
<td>0.7 6.54 4.91 3.67 2.97 0.00 0.34 0.51 0.55</td>
</tr>
<tr>
<td>$T = 10$</td>
<td>0.8 7.51 3.40 2.36 1.84 0.05 1.00 1.10 1.04</td>
<td>0.8 7.51 3.40 2.36 1.84 0.05 1.00 1.10 1.04</td>
</tr>
<tr>
<td>$T = 15$</td>
<td>0.9 8.06 1.15 0.82 0.70 3.21 3.77 3.17 2.57</td>
<td>0.9 8.06 1.15 0.82 0.70 3.21 3.77 3.17 2.57</td>
</tr>
<tr>
<td>$T = 0.99$</td>
<td>0.00 0.41 0.62 0.67 16.06 11.26 8.00 6.09</td>
<td>0.00 0.41 0.62 0.67 16.06 11.26 8.00 6.09</td>
</tr>
</tbody>
</table>
if the potential future benefits of learning that supplier 2 has a higher reliability outweigh the increase in current cost. In this case, to minimize the cost of learning, the firm prefers to order the lowest possible quantity from supplier 2. Therefore, the lower the minimum order quantity, the more the firm can afford to explore supplier’s reliability even if the current estimate is low.

4.3.2. Selecting a Second Supplier from Multiple Available Suppliers. Many firms maintain two suppliers for any given component to protect against supply uncertainties. Firms often select these two suppliers from an initial pool of potential suppliers. In the context of this paper, the firm might have multiple possible suppliers available to it when selecting “supplier 2” at the start of the horizon. In this section, we assume that once a selection is made, the firm can source only from supplier 1 and the chosen option for supplier 2; the other second-supplier options are unavailable during the horizon if not initially selected. In what follows, we identify the directional effects on parameters that induce the firm to select option for supplier 2; the other second-supplier option is low.

We next consider the influence of the reliability forecast distribution. In other words, how is the firm’s supplier preference influenced by the initial reliability forecasts? In particular, we consider how the forecast mean, $\bar{\theta}$, and SCV, $v$, influence the preference. The main result follows directly from the following technical lemma regarding the optimal value function when it is parameterized by $(\bar{\theta}, v)$.

**Lemma 2.** For $T \leq 3$, $V_T(\bar{\theta}, v)$ is decreasing in both $\bar{\theta}$ and $v$.

**Corollary 2.** All else being equal, for $T \leq 3$, supplier option $i$ is preferred to supplier option $j$ if $(\bar{\theta}_i, v_i) \geq (\bar{\theta}_j, v_j)$, where $(\bar{\theta}_i, v_i) \geq (\bar{\theta}_j, v_j)$ implies that $\bar{\theta}_i \geq \bar{\theta}_j$ and $v_i \geq v_j$.

In other words, if choosing between equal-cost options, the firm prefers options with higher forecast means and higher relative forecast uncertainties.

For a fixed forecast SCV, the fact that higher forecast means are more attractive is somewhat intuitive as it reflects that increasing reliability is attractive. The fact that higher relative forecast uncertainties are attractive for a fixed forecast mean is less intuitive. In the Bayesian updating procedure, the firm’s forecast is revised based on yield successes or failures. Let the updated mean after a success be denoted by $\bar{\theta}_S$ and the updated mean after a failure be denoted by $\bar{\theta}_F$. Consider two supplier options $i$ and $j$ that have the same forecast mean $\bar{\theta}$ but different forecast SCVs; let $i$ denote the supplier option with the higher SCV. The probability of a yield success is equal to $\bar{\theta}$ for both $i$ and $j$. However, the impact of a success or failure is more significant for option $i$ than $j$; that is, $\bar{\theta}_S \geq \bar{\theta}_S^i$ and $\bar{\theta}_F \leq \bar{\theta}_F^i$. In other words, while the probability of the forecasted mean being revised upward or downward is equal for both suppliers, the degree to which $i$’s mean is revised upward or downward is larger than for $j$’s mean. In and of itself, this does not fully explain why higher SCV options are more attractive, because the disadvantage of a more

---

2 We assume that $C_{\bar{\theta}}(\theta^*, \theta)$ is decreasing in $\theta^*$ (which is true if $v \geq g^2$) in the proof for $T = 3$. This is a sufficient but not necessary condition in the proof for $T = 3$. Extensive numerical testing suggests that Lemma 2 holds for all $T$ and even if $C_{\bar{\theta}}(\theta^*, \theta)$ is increasing in $\theta^2$. Corollary 2 is a direct consequence of Lemma 2, so Corollary 2 will hold if Lemma 2 holds. We also note that using Lemma 2, one can establish the following result for $T \leq 4$: In any period $t = T, \ldots, 1$, if it is optimal, for a given forecast mean, to source from supplier 2 when the SCV is $v_j$, then it is optimal to source from supplier 2 for any forecast with the same mean but a higher SCV, i.e., $v > v_j$. 


severe downward revision might outweigh the benefits of a larger upward revision. It turns out, however, that there are increasing returns to reliability; that is, the optimal single-period expected cost is concave decreasing in a supplier’s reliability. Because of this, the benefits of a higher upward reliability revision dominates the disadvantage of a downward reliability revision, and therefore suppliers with higher SCVs are favored.

What if, all else being equal, the firm must choose between supplier options $i$ and $j$ when the reliability forecasts $(\tilde{\theta}_i, \upsilon_i)$ and $(\tilde{\theta}_j, \upsilon_j)$ are not ordered such that one forecast has both a higher mean and higher SCV? In other words, what is the trade-off between the forecast mean and the SCV? We explore this question numerically. Using the same parameters as for Figure 2, Figure 4 presents iso-cost contours, i.e., constant $VT(\theta, \upsilon)$, for $T = 5$, $10$, and $15$. The contour value was chosen so that the average cost per period was the same for each value of $T$. First, we note that, as one would expect from Lemma 2, the contours are downward sloping; that is, a reduction in the mean must be offset by an increase in the SCV. Furthermore, they are convex decreasing; in other words, one needs increasingly larger increases in the SCV to offset larger reductions in the mean. Finally, we note that the iso-cost curve is lower for higher values of $T$, reflecting the fact that higher forecast SCVs are more valuable for larger $T$ because there is more time to potentially learn that the supplier has a higher-than-expected reliability, and also more to gain if the reliability is, in fact, higher.3

5. Impact of Reliability Uncertainty on Inventory Investment

If inventory can be carried from one period to the next, then a firm may use inventory instead of, or in addition to, dual sourcing to mitigate supply risk. Inventory storage enables the firm to make multiple attempts at procuring the necessary units to fill some future demand (Henig and Gerchak 1990). If the order succeeds, then it can be stored for future use; but if it fails, then the firm can try again in the following period. In this section, we investigate how uncertainty about supplier reliability influences the use of inventory to mitigate supply failures. We use the same Bayesian forecast-updating approach as used in the previous section. To focus on the inventory tactic rather than on dual sourcing, we assume that the firm has a single supplier and so simply refer to the “supplier” and do not superscript supplier variables. We first note that because any left-over inventory must be salvaged (or disposed of) at the end of the horizon, the final-period analysis is identical to that of §4 tailored to a single supplier case. All the final-period results of §4 when $\theta^1 < \gamma^1_A$, i.e., supplier 1 is not viable (see §A1), apply to the final-period of the finite-horizon, single-supplier, problem when inventory carrying is allowed.

Our goal is to develop insights into the impact of reliability uncertainty on the use of inventory rather than to develop an implementable decision-support tool. As such, we use a stylized two-period model. To be consistent with the previous section we continue to index time in a backward manner, so period 1 is the final period. Inventory carried over from period 2 to period 1 costs $h$ per unit. Unfilled demand at the end of either period is lost at a per-unit cost of $\pi$. We assume that demand is deterministic because that allows us to focus on the use of inventory as it relates to supply uncertainty. For ease of exposition, we also impose the following two restrictions: The minimum order quantity $m$ is less than or equal

3 We note that the curve for $T = 5$ is very close to, but is still lower than, the curve for the maximum possible SCV, i.e., $\tilde{\theta}/(1 - \tilde{\theta})$.
to the demand \( d \), and there is no discounting, that is \( \delta = 1 \). We note that each of these assumptions (lost sales, deterministic demand, minimum order-quantity size, and no discounting) can be relaxed without altering the primary insight (presented later) regarding the impact of reliability uncertainty on the use of inventory.

If in period 2 the firm orders more than demand \( d \), and the order succeeds, then it will carry inventory from period 2 to period 1. The only motive for carrying said inventory is to protect against a possible supply failure in period 1. We first characterize the firm’s optimal order quantity \( q^*_2(\theta) \) for the case of perfect supplier reliability information, because this serves as the benchmark for investigating the impact of reliability uncertainty on the firm’s inventory investment. We note that the optimal final-period order is characterized during the proof of the following theorem.

**Theorem 6.** If the supplier’s reliability \( \theta \) is known with certainty, then

\[
q^*_2(\theta) = \begin{cases} 
0, & \theta < \frac{c}{\pi - g}; \\
 d, & \frac{c}{\pi - g} \leq \theta \leq 1; \\
\frac{\pi + c - g - h}{2(\pi - g)} \sqrt{\left(\frac{\pi + c - g - h}{2(\pi - g)}\right)^2 - \frac{c}{\pi - g}}, & \frac{c}{\pi - g} \leq \theta \leq \frac{\pi + c - g - h}{2(\pi - g)}; \\
2d, & \theta < \frac{\pi + c - g - h}{2(\pi - g)} + \sqrt{\left(\frac{\pi + c - g - h}{2(\pi - g)}\right)^2 - \frac{c}{\pi - g}}; \\
\frac{\pi + c - g - h}{2(\pi - g)} + \sqrt{\left(\frac{\pi + c - g - h}{2(\pi - g)}\right)^2 - \frac{c}{\pi - g}}, & \frac{\pi + c - g - h}{2(\pi - g)} \leq \theta \leq 1; \\
\end{cases}
\]

if \( h \geq \pi + c - g - 2\sqrt{c(\pi - g)} \), and

If the firm orders \( 2d \) in period 2 and the order succeeds, then the firm is completely protected from failures in period 1 because it has sufficient inventory on hand to meet all demand. From the above theorem, we see that the firm pursues this inventory strategy, i.e., \( q^*_2(\theta) = 2d \), only if (i) inventory is not too expensive and (ii) the supplier reliability \( \theta \) is not too high or too low. If the supplier is highly reliable, then there is no incentive to invest in inventory for the future. If the supplier is quite unreliable, then the benefit of having future inventory is outweighed by the probability that the order will fail and the fact that the firm incurs the per unit order cost \( c \) even if the order fails.

We now turn to the case where the firm is uncertain about the supplier’s reliability. The firm begins with a forecast of the supplier’s reliability. The forecast has a Beta distribution and is parameterized by \((\alpha, \beta)\). As mentioned above, the optimal final-period policy is identical to that developed in §4 tailored to the case of a single supplier, i.e., \( \theta^1 < \gamma^1 \). The following theorem characterizes the optimal first-period order quantity \( q^*_2(\alpha, \beta) \).

**Theorem 7.** If \( \alpha/(\alpha + \beta) > c/(\pi - g) \), then \( q^*_2(\alpha, \beta) = 2d \) if

\[
c + \left(\frac{\alpha}{\alpha + \beta}\right)(h + g - \pi + \left(\frac{\alpha + 1}{\alpha + \beta + 1}\right)(\pi - g)) < 0,
\]

and \( q^*_2(\alpha, \beta) = d \) otherwise. If \( \alpha/(\alpha + \beta) \leq c/(\pi - g) \), then \( q^*_2(\alpha, \beta) = m \) if

\[
cm - \left(\frac{\alpha}{\alpha + \beta}\right)(\pi - g)m + \left(\frac{\alpha + 1}{\alpha + \beta + 1}\right)(\pi - c)d < 0,
\]

and \( q^*_2(\alpha, \beta) = 0 \) otherwise.

While the \((\alpha, \beta)\) forecast parametrization is useful for characterizing the optimal order quantity, it is more informative to discuss the implications of this theorem by using the mean and SCV parametrization \((\tilde{\theta}, v)\).

**Corollary 3.** For a given reliability forecast mean \( \tilde{\theta} \), if the firm invests in inventory, i.e., \( q^*_2(\tilde{\theta}, v) = 2d \), when the forecast SCV is \( v_\alpha \), then the firm will invest in inventory for any forecast with a lower SCV, i.e., for all \( v < v_\alpha \), but it might not be optimal for the firm to invest inventory for some \( v > v_\alpha \).
Interestingly, the firm is less willing to invest in inventory when it less certain about the supplier’s reliability. That is, the higher the forecast SCV, the smaller the range of forecast means for which the firm will invest in inventory to protect against a possible supply failure in the following period. In essence, reliability uncertainty makes inventory a less attractive failure-mitigation tactic. The reason is that investing in inventory to protect against future supply failures benefits the firm only if that inventory actually arrives, i.e., the supplier succeeds in filling the current order. If the supplier succeeds, however, then the firm’s estimate of the supplier’s reliability is revised upward, which in turn reduces the firm’s concerns about future supply risk, thus reducing the benefit of the inventory. In short, the successful arrival of the inventory from the supplier reduces the firm’s perceived need for that inventory. We note that the underlying supplier reliability does not change but the firm’s estimate of that reliability does, and it is the firm’s estimate that determines its perceived need for inventory.

6. Extensions

We now relax a number of assumptions to explore which results continue to hold under more general models. In particular, we extend our model to allow for general yield distributions, a general number of suppliers, and inventory carryover in the two-supplier case.

6.1. General Yield Distribution

To this point, we have assumed that supplier yields have Bernoulli (all-or-nothing) distributions. In this section, we extend our model to the case of general, discrete yield distributions. Supplier yields are assumed to be independent but need not be identical. The firm receives a random fraction \(0 \leq \bar{Y}_i \leq 1\) of the order placed with supplier \(i\), where \(\bar{Y}_i\) can take on one of \(K_i\) values, \(y_{i1}, \ldots, y_{iK_i}\), with associated probabilities \(\theta_{i1}, \ldots, \theta_{iK_i}\). We summarize the yield distribution for supplier \(i = 1, 2\) by the pair of vectors \((y_i', \theta')\), where \(y_i' = (y_{i1}, \ldots, y_{iK_i})\) and \(\theta' = (\theta_{i1}, \ldots, \theta_{iK_i})\).

We first consider the finite-horizon model of §4. If the firm has perfect knowledge of both suppliers’ yield distributions, then, as before, the finite-horizon problem collapses to a series of identical single-period problems. Let \(\tilde{C}(q; y', \theta, y^2, \theta^2)\) denote the single-period expected cost as a function of the order vector \(q = (q^1, q^2)\), where, for ease of notation, we have dropped the parentheses around the \((y', \theta')\) pairs. In the general-yield case, (2) becomes

\[
\tilde{C}(q; y^1, \theta^1, y^2, \theta^2) = \frac{2}{k^1 k^2} \bar{C}(y^1 q^1 + \sum_{k=1}^{k^1} \theta_{i1} y_{ik}^1 q^1 + y_{iK_i}^1 q^1) + \sum_{k=1}^{k^1} \sum_{l=1}^{k^2} \sum_{j=1}^{k^2} \theta_{ij}^1 \theta_{lj}^2 L(y_{ik} y_{lj}^1 + y_{iK_i} y_{lj}^1 y_{iK_i}^2 q^2). \tag{8}
\]

As before, we assume that the firm has perfect knowledge of supplier 1’s yield distribution but imperfect knowledge of supplier 2’s distribution. In particular, we assume that the firm knows the possible yield realizations \(y^2\) but has only a forecast of the associated probabilities \(\theta^2\). We denote this forecast by the joint density \(h(\theta^2)\).

In the single-period case, the firm’s problem is to minimize \(E_w[\tilde{C}(q; y^1, \theta^1, y^2, \theta^2)]\) subject to \(q \in Q\). From Equation (8) we see that \(\tilde{C}(q; y^1, \theta^1, y^2, \theta^2)\) is linear in \(\theta^2\), and therefore,

\[
E_w[\tilde{C}(q; y^1, \theta^1, y^2, \theta^2)] = \tilde{C}(q; y^1, \theta^1, y^2, E(\theta^2)).
\]

As before, we have a certainty equivalence result; the optimal solution to the single-period, uncertain-reliability problem can be found by solving the single-period, perfect-information problem with supplier 2’s yield probabilities set to their forecast means.

For the general finite-horizon problem, we assumed that the reliability forecast, i.e., the forecast of the success probability \(\theta^2\), had a Beta distribution. The Dirichlet distribution is the multivariate extension of the Beta distribution and, as such, is a natural choice\(^4\) for the joint density \(h(\theta^2)\). Therefore,

\[
h(\theta^2) = \frac{\Gamma\left(\sum_{k=1}^{K^2} \alpha_k\right)}{\prod_{k=1}^{K^2} \Gamma(\alpha_k)} \prod_{k=1}^{K^2} (\theta^2_k)^{\alpha_k-1},
\]

where \(\alpha = (\alpha_1, \ldots, \alpha_{K^2}) > 0\). We will denote the Dirichlet distribution by \(D(\alpha)\). Let \(\bar{\alpha}_k = \alpha_k / (\sum_{i=1}^{K^2} \alpha_i)\) and \(\bar{\alpha} = (\bar{\alpha}_1, \ldots, \bar{\alpha}_{K^2})\). We note that \(E(\theta^2) = \bar{\alpha}\). For \(k = 1, \ldots, K^2\), let \(e_k\) denote the unit vector with element \(k\)

\(^4\) In a working paper that builds upon an earlier version of this paper, Chen et al. (2007) also adopt a Dirichlet distribution for supply updating but do so in the context of supply disruptions.
equal to 1 and the other $K^2 - 1$ elements all equal to zero. If the prior distribution for $\theta^i$ is $D(\alpha)$, then the posterior distribution will be $D(\alpha + e_i)$ if yield event $y_i$ occurs. The probability (given the firm’s beliefs) of this yield event is $\tilde{\alpha}_i$. The state variable for the firm’s sourcing dynamic program will be the $\alpha$ vector. We note that this Dirichlet/general-yield model recovers the Beta/Bernoulli model of §4 if $K^i = 2$ and $y^i = (0, 1)$ for $i = 1, 2$. (As is the convention for Beta distribution, the $\alpha$ vector was denoted by $(\alpha_1, \alpha_2)$ rather than $(\alpha_1, \alpha_2)$ in the Beta/Bernoulli model.)

Analogous to the Bernoulli-yield case, see (5) and (6), define the following two order vectors:

$$q^{S2}(y^1, \theta^1, y^2, \theta^2) = \arg\min_{q \in Q^{S2}} \{\tilde{C}(q; y^1, \theta^1, y^2, \theta^2)\},$$

$$q^{N2}(y^1, \theta^1) = \arg\min_{q \in Q^{N2}} \{\tilde{C}(q; y^1, \theta^1, y^2, \theta^2)\}.$$

Also, define $P_t$ as the set of states $\alpha$ for which it is optimal to source from supplier 2 in period $t$. The following theorem generalizes Theorems 1 and 2 to the case of general yield distributions and establishes that the basic structure of the optimal policy is preserved.

**Theorem 8.** For period $t = T, \ldots, 1$, define $q^i_t(\alpha)$ as the optimal order vector in period $t$.

(i) For any period $t = T, \ldots, 1$, $q^i_t(\alpha) = q^{S2}(y^1, \theta^1, y^2, \tilde{\alpha})$ if $\alpha \in P_t$ and $q^i_t(\alpha) = q^{N2}(y^1, \theta^1)$ otherwise.

(ii) $P_{t+1} \subseteq P_t$ for $t = T, \ldots, 2$.

In §4.3.2, we considered the situation in which the firm selects the second supplier from an initial pool of potential suppliers, and we established a number of results regarding the firm’s preference amongst suppliers. The following theorem extends some of our earlier results. As before, subscripts on parameters denote a particular second-supplier option in what follows.

**Theorem 9.** All else being equal, supplier-option $i$ is preferred to supplier-option $j$ if $c_i^2 < c_j^2$, if $s_i^2 < s_j^2$, if $m_i^2 < m_j^2$, or if $y_i^2 > y_j^2$.

The cost and minimum-order quantity preference results are identical to the Bernoulli yield case. The yield-event preference result establishes that, as one would expect, the firm prefers a supplier with a stochastically higher yield. This is somewhat analogous to the expected yield result for the Bernoulli case. However, in the general yield case, a higher expected yield does not, in and of itself, ensure a stochastically higher yield.

We now turn our attention to the single-supplier, two-period, inventory-carryover model of §5. The analysis for the general yield two-period model is more complex than that for the Bernoulli yield model, and we do not include the analysis here because of space. For a Bernoulli all-or-nothing yield case, we established that the firm is less likely to invest in inventory when it is uncertain about the supplier’s reliability. The underlying reason for this is that inventory is available in the second period only if the supplier succeeds in the first period, but that this very success reduces the perceived value of second-period inventory as it causes the firm to revise (upward) its estimate of the supplier’s reliability. In the general yield case, however, one can construct problem instances for which the firm, in fact, invests in more inventory in the first period if it is uncertain about the supplier’s yield distribution than it does if it is certain. The reason is that with uncertainty about the distribution, a low yield in the first period causes the firm to revise the distribution such that it now believes that there is a higher probability of a low yield than it originally thought. This increases the perceived value of second-period inventory, just as a high first-period yield realization reduces the value. In the all-or-nothing yield model, a low yield is a failure; therefore, while second-period inventory is more valuable in that event, no second-period inventory is, in fact, available. However, if a low-yield scenario results in a positive portion being delivered, some second-period inventory might be available if the firm orders a sufficiently large quantity in the first period. This inventory is more valuable under imperfect yield information because the updated yield distribution is less favorable than the original; therefore, one can construct instances in which the firm orders a higher quantity when it is uncertain about the supplier’s yield distribution.

The problem instances that we identified (for which the firm invests in more inventory when it is uncertain about the yield distribution) were ones in which the supplier was very uncertain about the supplier’s yield distribution and in which inventory was inexpensive relative to lost sales.
about the yield distribution, the more significant the distribution revision in the event of a low-yield scenario, so inventory has more potential value. However, inventory should be cheap if the firm is going to purchase enough units in the first period to have left-over inventory in the event of a low yield.

6.2. General Number of Suppliers

We now extend the finite-horizon model of §4 to allow for a general number (N) of suppliers. Recall that the firm can choose whether or not to order from a supplier, but that if it does order from a supplier, it must order a quantity greater or equal to that supplier’s minimum order quantity. Directly analogous to (1), we define the following sets:

\[ Q_m^i = \{ q^i \in \mathbb{R} : q^i \geq m^i \}, \quad i = 1, \ldots, N, \]

\[ Q = Q_m^i \cup \{0\}, \quad i = 1, \ldots, N, \]

\[ Q = \{ q \in \mathbb{R}^N : q^i \in Q^i, \quad i = 1, \ldots, N \}. \]

That is, \( Q^i \) is the set of feasible orders from supplier \( i = 1, \ldots, N \), and \( Q \) is the set of feasible order vectors \( q = (q^1, \ldots, q^N) \). In what follows, we consider the general yield model as presented in the preceding section, with the understanding that this model includes the Bernoulli yield model as a special case. In the perfect yield information case, let \( \hat{C}(q; y^1, \theta^1, \ldots, y^N, \theta^N) \) denote the single-period expected cost as a function of the order vector \( q = (q^1, \ldots, q^N) \in Q \). Then, recalling that supplier yields are independent,

\[
\hat{C}(q; y^1, \theta^1, \ldots, y^N, \theta^N) = \sum_{i=1}^N \left( c^i q^i + \sum_{k=1}^{K^i} \theta_k^i \gamma_k q^i \right) + \sum_{k_1=1}^{K^1} \sum_{k_2=1}^{K^2} \cdots \sum_{k_N=1}^{K^N} \left( \prod_{i=1}^N \theta_k^i \right) L \left( \sum_{i=1}^N \gamma_k^i q^i \right). 
\]

We assume that the firm has perfect knowledge of the yield distributions for suppliers \( i = 1, \ldots, N_p \), but imperfect knowledge for suppliers \( i = N_p + 1, \ldots, N \), where \( 0 \leq N_p \leq N \). The case of \( N_p = N \) (\( N_p = 0 \)) reflects the situation in which the firm has perfect knowledge about all (none) of the suppliers’ yield distributions. As before, for suppliers \( i = N_p + 1, \ldots, N \), we assume that the firm knows the possible yield realizations \( y^i \) but has only a forecast of the associated probabilities \( \theta^i \). We denote the forecast for supplier \( i = N_p + 1, \ldots, N \) by the joint density \( h^i(\theta^i) \). We assume that the forecasts are independently distributed across all suppliers. Because of this independence,

\[
E_{\theta^{N_p+1}, \ldots, \theta^N} \left[ \hat{C}(q; y^1, \theta^1, \ldots, y^N_p, \theta^N_p, \right. \\
\left. y^{N_p+1}, \theta^{N_p+1}, \ldots, y^N, \theta^N) \right] \\
= \hat{C}(q; y^1, \theta^1, \ldots, y^{N_p}, \theta^{N_p}, \\
\left. y^{N_p+1}, E_{\theta^{N_p+1}}[\theta^{N_p+1}], \ldots, y^N, E_{\theta^N}[\theta^N] \right),
\]

and again we have a certainty equivalence result for the single-period problem.

For the finite-horizon problem, we assume that the yield forecast for supplier \( i = N_p + 1, \ldots, N \) has a Dirichlet distributions, \( D(\alpha) \). We first consider the case in which \( N_p = N - 1 \), i.e., the firm has perfect information of all yield distributions except for that of supplier \( N \). Then it is straightforward to show that all of our two-supplier results for reliability uncertainty (i.e., Theorems 1 to 5, Lemmas 1 and 2, Corollaries 1 and 2) as well as the general-yield results (i.e., Theorems 8 and 9) continue to hold for this \( N \)-supplier case, with supplier 2 being replaced by supplier \( N \). To see this, analogous to (4), define the sets

\[
Q^N_{SN} = \{ q \in \mathbb{R}^N : q^i \in Q^i, i = 1, \ldots, N - 1, q_N \in Q_m^N \},
\]

\[
Q^N_{NN} = \{ q \in \mathbb{R}^N : q^i \in Q^i, i = 1, \ldots, N - 1, q_N = 0 \},
\]

and, analogous to (5) and (6), define the order vectors

\[
q^N_{SN}(y^1, \theta^1, \ldots, y^N, \theta^N) = \text{arg min}_{q \in Q^N_{SN}} \left\{ \hat{C}(q; y^1, \theta^1, \ldots, y^N, \theta^N) \right\},
\]

\[
q^N_{NN}(y^1, \theta^1, \ldots, y^{N-1}, \theta^{N-1}) = \text{arg min}_{q \in Q^N_{NN}} \left\{ \hat{C}(q; y^1, \theta^1, \ldots, y^N, \theta^N) \right\}.
\]

Letting

\[
\hat{C}^*_{SN}(y^1, \theta^1, \ldots, y^N, \theta^N) = \hat{C}(q^N_{SN}; y^1, \theta^1, \ldots, y^N, \theta^N),
\]

\[
\hat{C}^*_{NN}(y^1, \theta^1, \ldots, y^{N-1}, \theta^{N-1}) = \hat{C}(q^N_{NN}; y^1, \theta^1, \ldots, y^N, \theta^N),
\]

then all our earlier results, mentioned above, go through directly with \( q^N_{SN}(\cdot), q^N_{NN}(\cdot), \hat{C}^*_{SN}(\cdot), \) and \( \hat{C}^*_{NN}(\cdot) \) replacing \( q^{SN}(\cdot), q^{NN}(\cdot), \hat{C}_S(\cdot), \) and \( \hat{C}_S(\cdot) \), respectively.
We now turn our attention to the more general case in which $N_p < N - 1$, that is, the firm is uncertain about more than one supplier’s yield distribution. In this case, the state for the firm’s finite-horizon sourcing problem is the set of $\alpha^i$ vectors for suppliers $i = N_p + 1, \ldots, N$, which can be represented by a matrix $\Lambda$ if one appropriately accounts for the fact that the $\alpha^i$ may have different lengths.\(^5\) Let $\Omega \subseteq \{1, \ldots, N\}$ denote any arbitrary subset (including the null set) of suppliers. We say that $\Omega_t^*(\Lambda)$ is the active set of suppliers at time $t$ for state $\Lambda$ if it is optimal to source a positive quantity, i.e., $q_i \geq m_i$, from all suppliers $i \in \Omega_t^*(\Lambda)$ and to source nothing, i.e., $q_i = 0$, from any supplier $j \notin \Omega_t^*(\Lambda)$. We say that the remaining suppliers are inactive at time $t$. Define $P_t$ as the set of states $\Lambda$, at time $t$, for which at least one supplier from $\{N_p + 1, \ldots, N\}$ is active, i.e., $\Lambda \in P_t \iff \exists i \in \{N_p + 1, \ldots, N\}$ such that $i \in \Omega_t^*(\Lambda)$.

**Theorem 10.** $P_{t-1} \subseteq P_t$ for $t = T, \ldots, 2$.

This $N$-supplier result is analogous to Theorem 2 for the two-supplier case but the implications are less strong.\(^6\) Similar to Theorem 2, for which $N = 2$ and $N_p = 1$, it implies that if all suppliers in $\{N_p + 1, \ldots, N\}$ are inactive at time $t$, then all of these suppliers will remain inactive for the rest of the horizon. However, it does not imply that any particular supplier in $\{N_p + 1, \ldots, N\}$ will remain inactive for the rest of the horizon if that supplier is inactive at time $t$ (as was the case when there was distributional uncertainty about only one supplier). When there is distributional uncertainty about more than one supplier, then the state $\Lambda$ continues to evolve if at least one supplier from $\{N_p + 1, \ldots, N\}$ is active at time $t$, although only the rows (i.e., the $\alpha_i$) for the active suppliers in $\{N_p + 1, \ldots, N\}$ evolve. As the state evolves, then a supplier that was inactive earlier may become active later. Intuitively, this could occur if the firm observes low yields from some active suppliers. Then as the yield distributions are updated to reflect performance, a previously inactive supplier may supplant one or more of the poorly performing suppliers. Of course, how long the newly added supplier may remain active will depend on its performance and that of the other active suppliers. Characterizing the evolution of the active set is the subject of future research.

### 6.3. Two Suppliers and Inventory Carryover

In §4 we allowed for two suppliers but no inventory carryover, whereas in §5 we allowed for inventory carryover (in a two-period model) but only one supplier. We now consider a version of §4 with inventory carryover. Inventory at the end of any period $t = T, \ldots, 2$ costs $h$ per unit. Let $y_t$ denote the inventory at the beginning of period $t = T, \ldots, 1$. We assume that demand in each period is deterministic, equal to $d$, and at least as large as supplier 2’s minimum order quantity, i.e., $d > m^2$. Furthermore, we assume that supplier 1 is perfectly reliable, i.e., $\theta_1 = 1$, imposes no minimum order quantity, i.e., $m^1 = 0$, and allows the firm to return up to $d$ units in a period for the purchase price. To avoid the trivial cases in which supplier 1 is always optimal or there is sufficient initial inventory to meet all demand over the horizon, we restrict attention to $c^1 + g^1 < \pi$ and $y_T < Td$.

Without inventory carryover, the state of the system at the start of a period was fully captured by the current reliability forecast ($\alpha, \beta$). With inventory carryover, the state\(^7\) must now include the current inventory $y$. Define $P_t^y$ as the set of reliability forecasts ($\alpha, \beta$) for which it is optimal to source from supplier 2 in period $t$ if the current inventory is $y$.

**Theorem 11.** $P_{t-1}^y \subseteq P_t^y$ for $t = T, \ldots, 2$.

In other words, for the same inventory level and reliability forecast, if it is optimal to source from supplier 2 in period $t$, then it is optimal to source from supplier 2 in any earlier period. This is directly analogous to Theorem 2 for the case of no inventory carryover. However, in that case, if it was optimal not to...

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\(^5\) For notational convenience, let $N_c = N - N_p$, i.e., $N_p$ is the number of suppliers for which the firm is uncertain about their yield distributions, and let $K_{\text{max}} = \max_{j=1}^{N_p} \{K^{\text{v}^+}\}$. Let us define the matrix $\Lambda$ of size $N_c \times K_{\text{max}}$ such that element $[\Lambda]_{j,k} = \alpha^i_{j,k} - \beta^i_{j,k}$ for $j = 1, \ldots, N_c$ and $k = 1, \ldots, K^{\text{v}^+}$ and $[\Lambda]_{j,k} = 0$ if $K^{\text{v}^+} < k < K_{\text{max}}$. In other words, the rows of $\Lambda$ contain the $\alpha^i$ vectors for $i = N_p + 1, \ldots, N$, but the rows also, as necessary, contain some zero elements in recognition of the fact that the $\alpha^i$ may have different lengths. The system state is then represented by the matrix $\Lambda$.

\(^6\) For completeness, a generalization of Theorem 1 is contained in the proof of Theorem 10 but omitted here because of space.

\(^7\) For space, the development of and expressions for the optimal value function are contained in the proof of Theorem 11.
source from supplier 2 in period $t$, then it was never optimal to source from supplier 2 in any later period. This is not the case with inventory carryover. It might not be optimal to source from either supplier at time $t$ if the firm has lots of inventory, but as the inventory depletes over time, it might be optimal to source from supplier 2 at a later time. We note, however, that if it is not optimal to source from supplier 2 when the inventory is zero, then it will never again be optimal to source from supplier 2.

7. Conclusions
A pervasive but implicit assumption in the supply-uncertainty literature is that the firm knows supply distributions with certainty. We argue that this is a strong assumption in many circumstances. A firm is more likely to have a forecast of a supplier’s yield distribution and to update that forecast based on its experiences with the supplier. Demand learning, whereby a firm updates its estimate of the demand distribution based on demand or sales realizations, has received extensive attention in the operations literature. To the best of our knowledge, ours is the first paper to consider the operational implication of supply learning.

Introducing a Bayesian approach to supply learning, we characterize the optimal, finite-horizon, operating policy (sourcing and/or inventory) for a firm with unreliable suppliers. We establish that supply learning can have a significant impact on the operating policy. In particular, in the case of Bernoulli, all-or-nothing yields, an increase in the firm’s uncertainty about a supplier’s reliability makes that supplier more attractive because of the potential gains from learning that the supplier has a higher reliability than initially thought. Increasing reliability uncertainty makes the firm less willing to invest in inventory because inventory is beneficial only if it arrives, and the arrival of the inventory increases the firm’s estimate of the supplier’s reliability, which in turn reduces the firm’s perceived need for said inventory.

We view this paper as an important first step in understanding the operational implications of supply learning, but it is a first step, so there is ample scope for future work in this area. For example, future work could further develop and explore the general number-of-supplier and inventory-carryover extensions presented in §6. Furthermore, the purpose of this work was primarily to understand the first-order effects of supply learning on sourcing and inventory decisions, and as such, the models are somewhat stylized in nature. We hope this work has laid a foundation for future work that embeds supply learning in more tactically oriented models.

Electronic Companion
An electronic companion to this paper is available on the Manufacturing & Service Operations Management website (http://msom.pubs.informs.org/ecompanion.html).

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