Technical Appendix: Multi-Product Firms and Trade Liberalization (Not For Publication)

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Contents

1. Introduction

This appendix contains the technical derivations of expressions for each section of the paper, the proofs of propositions and additional supplementary material. Sections 2.-6. correspond to the sections with the same title in the main paper.

2. Related Literature on Multi-Product Firms

No derivations required.

3. The Model

In Section 3.1., we give the expression for weighted average productivity. In Section 3.2., we consider a hybrid specification of the model where product attributes have both common and country-specific components, as discussed in footnote 7 in the paper. In Section 3.3., we extend the model to incorporate steady-state adding and dropping of products, as discussed in footnote 8 in the paper. In Section 3.4., we extend the model to incorporate multiple factors of production and multiple industries, as referred to in footnote 10 in the paper.

3.1. Weighted Average Productivity

Weighted average productivity depends on the firm ability cutoff (φ_{ij}^{*}) and a weighted average of product attributes for each firm ability ($\tilde{\lambda}_{ij}(\varphi)$):

$$\tilde{\varphi}_{ij} \equiv \left[\frac{1}{1 - G_i\left(\varphi_{ij}^{*}\right)} \int_{\varphi_{ij}^{*}}^{\infty} \left(\varphi \tilde{\lambda}_{ij}(\varphi)\right)^{\sigma-1} g_i(\varphi)\, d\varphi \right]^{\frac{1}{\sigma-1}}, \tag{1}$$

$$\tilde{\lambda}_{ij}(\varphi) = \left[\frac{1}{1 - Z\left(\lambda_{ij}^{*}(\varphi)\right)} \int_{\lambda_{ij}^{*}(\varphi)}^{\infty} \lambda^{\sigma-1} z(\lambda)\, d\lambda \right]^{\frac{1}{\sigma-1}}, \tag{2}$$

where $\tilde{\lambda}_{ij}(\varphi)$ depends on the product attributes cutoff for each firm ability ($\lambda_{ij}^{*}(\varphi)$).

3.2. Common and Country-Specific Components of Product Attributes

Consider a hybrid specification of the model, where product attributes have both common and country-specific components:

$$\lambda = \lambda_k^{\varsigma} \lambda_{jk}^{1-\varsigma}, \qquad 0 \leq \varsigma \leq 1$$

where λ_k is common across countries j for product k; λ_{jk} varies across both countries j and products k; and ς parameterizes the relative importance of the common and country-specific components.

Once the sunk entry cost is paid, a firm observes the common component of product attributes for each product, λ_k, and the country-specific component of product attributes, λ_{jk}, for each product and country. The common component is drawn separately for each product from a continuous distribution $z_k(\lambda_k)$ with cumulative distribution function $Z_k(\lambda_k)$. The country-specific component is drawn separately for each product and country from a continuous distribution $z_{jk}(\lambda_{jk})$ with cumulative distribution function $Z_{jk}(\lambda_{jk})$.

To make use of law of large numbers results, we assume that the firm ability and product attributes distributions are independent of one another and independently distributed across firms. Similarly, we assume that the common product attributes distribution, $z_k(\lambda_k)$, is independently distributed across products, while the country-specific product attributes distribution, $z_{jk}(\lambda_{jk})$, is independently distributed across products and countries. With these assumptions, a firm's profitability is now correlated across products and countries for two reasons. First, higher ability (φ) raises a firm's profitability across all products and countries. Second, a higher common component of product attributes (λ_k) raises a firm's profitability across all countries for a given product. These correlations are however imperfect because of stochastic variation in the country-specific component of product attributes. As $\varsigma \to 1$, the hybrid case reduces to the common-product-attributes specification discussed in the paper. As $\varsigma \to 0$, the hybrid case reduces to the country-specific-product attributes specification discussed in the paper.

3.3. *Steady-state Product Adding and Dropping*

As the focus of the paper is the cross-section distribution of exports across firms, countries and products, we follow much of the literature on firm heterogeneity in international trade in abstracting from dynamics. In this section, we show that the model can be extended to introduce stochastic variation in firm ability and product attributes over time. In this extension, we embed a simplified version of the dynamics from the closed economy model of Bernard, Redding and Schott (2010) in an open economy setting. While we consider symmetric countries and the country-specific-product-attributes specification, it is straightforward to instead consider asymmetric countries and the common-product-attributes specification.

The specification of entry, production and demand is similar to that considered in the paper, except that we consider a continuous time version of the model with steady-state entry and exit of firms and steady-state adding and dropping of products within firms. Once a firm incurs the sunk entry cost, f_e, firm ability and product attributes are drawn from the continuous distributions $g(\varphi)$ and $z(\lambda)$ respectively, with cumulative distributions $G(\varphi)$ and $Z(\lambda)$. After a firm observes its initial values of ability and product attributes, it decides whether to produce or exit. If the firm exits, its production knowledge is lost, and the sunk cost must be incurred again in order for the firm to re-enter. If the firm enters, it faces a Poisson probability $\theta > 0$ of a shock to ability φ, in which case a new value for ability φ' is re-drawn from the same distribution as upon entry $g(\varphi')$. The firm also faces a Poisson probability $\varepsilon > 0$ of an idiosyncratic shock to product attributes λ for its variety of a given product in a given country, in which case a new value for product attributes λ' is re-drawn from the same distribution as upon entry $z(\lambda')$.[1] As in Melitz (2003), the firm also faces a constant exogenous probability of death of $\delta > 0$, as a result of *force majeure* events beyond its control.

As product attributes for a firm's variety of each product change over time, previously profitable products and markets become unprofitable and are dropped when λ falls below the product attributes cutoff for the market $\{\lambda_d^*(\varphi),\ \lambda_x^*(\varphi)\}$. Similarly, previously unprofitable products become viable and are added when λ rises above the product attributes cutoff for the market $\{\lambda_d^*(\varphi),\ \lambda_x^*(\varphi)\}$. Since the distribution from which product attributes are drawn following a stochastic shock is the same as the distribution from which they are drawn upon entry, the stationary distribution for product attributes, $\gamma_z(\lambda)$, takes a simple form:

$$\gamma_z(\lambda) = z(\lambda), \tag{3}$$

where the stationary distribution for product attributes conditional on a product being supplied to a market is a truncation of $z(\lambda)$ at the product attributes cutoff for the market $\{\lambda_d^*(\varphi),\ \lambda_x^*(\varphi)\}$.

As the distribution from which firm ability is drawn following a stochastic shock is the same as the distribution from which it is drawn upon entry, the stationary distribution for firm ability, $\gamma_g(\varphi)$, also takes a simple form. The stationary distribution for firm ability conditional on supplying a market is a truncation of the distribution $g(\varphi)$ at the firm ability cutoff for serving the market:

$$\gamma_g(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi_d^*)} & \text{for } \varphi \geq \varphi_d^* \text{ in the domestic market} \\ \frac{g(\varphi)}{1-G(\varphi_x^*)} & \text{for } \varphi \geq \varphi_x^* \text{ in each export market} \\ 0 & \text{otherwise} \end{cases} . \tag{4}$$

The determination of general equilibrium remains largely the same as in the paper with a few appropriate modifications. In the dynamic extension considered here, the combination of a sunk entry cost and stochastic shocks to firm ability generates an option value to firm entry. If a firm chooses to exit, it forgoes both the net present value of its instantaneous flow of profits and also

[1] Assuming that firm ability and product attributes are re-drawn following a stochastic shock from the same distributions as upon entry is a simplifying device, which enables us to introduce dynamics in as tractable a way as possible. The closed economy model of Bernard, Redding and Schott (2010) considers richer forms of dynamics that allow for serial correlation in firm ability and product attributes. While introducing serial correlation complicates the model's dynamics, it does not change its cross-sectional predictions that are our focus.

the option of experiencing stochastic ability shocks. Therefore, with this option value to firm entry, the threshold ability for firm entry and exit will in general lie below the ability at which the instantaneous flow of total firm profits is equal to zero.[2] The value of a firm with ability φ is determined according to the following Bellman equation:

$$v(\varphi) = \frac{\pi(\varphi)}{\delta+\theta} + \left(\frac{\theta}{\delta+\theta}\right) \int_{\varphi_d^*}^{\infty} v(\varphi') \, g(\varphi') \, d\varphi'. \tag{5}$$

As the distribution from which a new value for ability is drawn following a stochastic shock is the same as upon entry and is independent of a firm's existing value for ability, the solution to this Bellman equation takes a simple form. Substituting for $v(\varphi')$ on the right-hand side of (5) using the trial solution $v(\varphi) = \alpha\pi(\varphi) + \beta \int_{\varphi_d^*}^{\infty} \pi(\varphi') \, g(\varphi') \, d\varphi'$, and solving for α and β, yields the equilibrium value of a firm:

$$v(\varphi) = \frac{\pi(\varphi)}{\delta+\theta} + \left(\frac{\theta}{\delta+\theta}\right) \left(\frac{1}{\delta+\theta G(\varphi_d^*)}\right) \left[\int_{\varphi_d^*}^{\infty} \pi(\varphi') \, g(\varphi') \, d\varphi'\right]. \tag{6}$$

Therefore the equilibrium value of a firm with ability φ is a weighted average of the current flow of firm profits and the expected flow of firm profits following a stochastic ability shock, where the weights depend on the probability of firm death, the probability of an ability shock and the probability that a firm remains active following an ability shock. Substituting the expression for the equilibrium value of a firm into the free entry condition, and rearranging, the free entry condition can be re-written as follows:

$$V = \int_{\varphi_d^*}^{\infty} \left[\frac{\pi(\varphi) + \left(\frac{\theta}{\delta+\theta}\right) \int_{\varphi_d^*}^{\infty} \left[\pi(\varphi') - \pi(\varphi)\right] g(\varphi') \, d\varphi'}{\delta+\theta G(\varphi_d^*)}\right] g(\varphi) \, d\varphi = f_e, \tag{7}$$

which can be simplified to yield the following expression:

$$V = \int_{\varphi_d^*}^{\infty} \left(\frac{\pi(\varphi)}{\delta+\theta G(\varphi_d^*)}\right) g(\varphi) \, d\varphi = f_e, \tag{8}$$

which can be in turn re-written as:

$$V = \int_{\varphi_d^*}^{\infty} \left(\frac{\pi_d(\varphi)}{\delta+\theta G(\varphi_d^*)}\right) g(\varphi) \, d\varphi + n \int_{\varphi_x^*}^{\infty} \left(\frac{\pi_x(\varphi)}{\delta+\theta G(\varphi_d^*)}\right) g(\varphi) \, d\varphi = f_e.$$

As in the paper, the model has a recursive structure, and the determination of general equilibrium is straightforward. We begin by determining $\{\varphi_d^*,\ \varphi_x^*,\ \lambda_d^*(\varphi_d^*),\ \lambda_x^*(\varphi_x^*)\}$ using four equilibrium relationships. To derive the first of these relationships, we use the expression for total firm profits

[2] In contrast, fixed and marginal production costs for individual products have no sunk component, and product attributes for a firm's variety of each product evolve independently of whether or not the variety is supplied. Therefore, once a firm has decided to enter, the firm's decision whether or not to supply each product reduces to a period-by-period comparison of contemporaneous revenue and production costs.

in the paper as well as $\lambda_d^*(\varphi) = (\varphi_d^*/\varphi)\,\lambda_d^*(\varphi_d^*)$ and $\lambda_x^*(\varphi) = (\varphi_x^*/\varphi)\,\lambda_x^*(\varphi_x^*)$, which together imply that the free entry condition (8) can be written as:

$$\begin{aligned} V = &\int_{\varphi_d^*}^{\infty}\left[\int_{(\varphi_d^*/\varphi)\lambda_d^*(\varphi_d^*)}^{\infty}\left(\left(\frac{\varphi\lambda}{\varphi_d^*\lambda_d^*(\varphi_d^*)}\right)^{\sigma-1}-1\right)f_d z(\lambda)\,d\lambda - F_d\right]\frac{g(\varphi)}{\delta+\theta G_c(\varphi_d^*)}d\varphi \\ &+\int_{\varphi_x^*}^{\infty}\left[\int_{(\varphi_x^*/\varphi)\lambda_x^*(\varphi_x^*)}^{\infty}\left(\left(\frac{\varphi\lambda}{\varphi_x^*\lambda_x^*(\varphi_x^*)}\right)^{\sigma-1}-1\right)f_x z(\lambda)\,d\lambda - F_x\right]\frac{ng(\varphi)}{\delta+\theta G_c(\varphi_d^*)}d\varphi = f_e. \end{aligned} \tag{9}$$

Since the only sunk cost in the model is the entry cost, f_e, and we consider an equilibrium with selection into export markets, the lowest ability firm that enters serves only the domestic market: $\varphi_d^* < \varphi_x^*$. Therefore only a firm's decision of whether or not to serve the domestic market is affected by the option value of entry. In contrast, the firm's decision whether or not to serve the export market is determined by a comparison of the instantaneous flow of revenue and the fixed exporting costs. The product and firm exporting cutoffs are therefore determined as in the paper:

$$\int_{\lambda_x^*(\varphi_x^*)}^{\infty}\left[\left(\frac{\lambda}{\lambda_x^*(\varphi_x^*)}\right)^{\sigma-1}-1\right]f_x z(\lambda)\,d\lambda = F_x \tag{10}$$

$$\varphi_x^* = \tau\left(\frac{f_x}{f_d}\right)^{\frac{1}{\sigma-1}}\left(\frac{\lambda_d^*(\varphi_d^*)}{\lambda_x^*(\varphi_x^*)}\right)\varphi_d^*. \tag{11}$$

In deciding whether or not to enter and supply the domestic market, a firm takes into account the option value of entry. At the domestic cutoff ability, φ_d^*, the value of the firm is equal to zero, $v(\varphi_d^*) = 0$, which from (6) implies that the flow of total firm losses exactly equals the probability of a stochastic shock to firm ability times expected profits conditional on a stochastic shock occurring:

$$\begin{aligned} &-\left[\int_{(\varphi_d^*/\varphi)\lambda_d^*(\varphi_d^*)}^{\lambda_d^*(\varphi_d^*)}\left(\left(\frac{\lambda}{\lambda_d^*(\varphi_d^*)}\right)^{\sigma-1}-1\right)f_p z(\lambda)\,d\lambda - F_d\right] \\ &= \int_{\varphi_d^*}^{\infty}\left[\int_{(\varphi_d^*/\varphi)\lambda_d^*(\varphi_d^*)}^{\lambda_d^*(\varphi_d^*)}\left(\left(\frac{\varphi\lambda}{\varphi_d^*\lambda_d^*(\varphi_d^*)}\right)^{\sigma-1}-1\right)f_d z(\lambda)\,d\lambda - F_d\right]\frac{\theta g(\varphi)}{\delta+\theta G(\varphi_d^*)}d\varphi \\ &+\int_{\varphi_x^*}^{\infty}\left[\int_{(\varphi_x^*/\varphi)\lambda_x^*(\varphi_x^*)}^{\lambda_x^*(\varphi_x^*)}\left(\left(\frac{\varphi\lambda}{\varphi_x^*\lambda_x^*(\varphi_x^*)}\right)^{\sigma-1}-1\right)f_x z(\lambda)\,d\lambda - F_x\right]\frac{n\theta g(\varphi)}{\delta+\theta G_c(\varphi_d^*)}d\varphi. \end{aligned} \tag{12}$$

To characterize $\{\varphi_d^*,\, \varphi_x^*,\, \lambda_d^*(\varphi_d^*),\, \lambda_x^*(\varphi_x^*)\}$, we first use (10) to determine $\lambda_x^*(\varphi_x^*)$ independent of the other equations of the model. Second, we use (11) to determine φ_x^* as a function of φ_d^*, $\lambda_d^*(\varphi_d^*)$ and $\lambda_x^*(\varphi_x^*)$. Third, substituting for φ_x^* and using the value of $\lambda_x^*(\varphi_x^*)$ determined above, (9) and (12) provide two equations that together determine $\{\varphi_d^*,\, \lambda_d^*(\varphi_d^*)\}$. Having characterized $\{\lambda_x^*(\varphi_x^*),\, \varphi_d^*,\, \lambda_d^*(\varphi_d^*)\}$, the equilibrium value of φ_x^* follows immediately from (11). Finally, having determined

$\{\varphi_d^*, \varphi_x^*, \lambda_d^*(\varphi_d^*), \lambda_x^*(\varphi_x^*)\}$, the remaining elements of the equilibrium vector can be determined as in the paper.

In this extension of the model in the paper, the general equilibrium features steady-state adding and dropping of products and destinations as well as steady-state entry and exit of firms. Each period a measure of new firms incur the sunk entry cost. Of these new firms, those with an ability draw above the domestic cutoff (φ_d^*) enter, while those with an ability draw below φ_d^* exit. Among incumbent firms, a firm with unchanged ability supplies constant measures of products to the domestic and export markets, but the identity of these products changes as stochastic shocks to product attributes occur. As a result of these stochastic shocks a firm with unchanged ability drops a measure of the products previously supplied to each market and adds an equal measure of the products not previously supplied to each market. Finally, as stochastic shocks to an incumbent firm's ability occur, the measure of products supplied to each market expands with increases in ability and contracts with decreases in ability. An incumbent firm enters export markets when its ability rises above the exporting cutoff (φ_x^*). An incumbent firm exits endogenously when its ability falls below φ_d^* or exogenously when death occurs as a result of *force majeure* considerations beyond the control of the firm.

While the extended model incorporates steady-state adding and dropping of products and destinations, the model's predictions for the cross-section distribution of exports across firms, products and countries remain unchanged. As these cross-sectional predictions are the focus of our analysis, we do not pursue this dynamic extension further in the paper.

3.4. Multi-Product Firms and Comparative Advantage

In this section of the appendix, we show that the model can be extended to incorporate comparative advantage based on cross-country differences in factor abundance and cross-industry differences in factor intensity. In this extension, we embed our model of multi-product firms within the two-factor, two-country and two-industry heterogeneous firm framework of Bernard, Redding and Schott (2007).

The model in the paper can be viewed as capturing a single industry containing many products, with firms supplying differentiated varieties of these products. We now generalize this framework by analyzing two industries, $i = 1, 2$, each of which has this structure. The two industries enter an upper tier of the representative consumer's utility that takes the Cobb-Douglas form:

$$\mathcal{U} = U_1^{\alpha_1} U_2^{\alpha_2}, \qquad \alpha_1 + \alpha_2 = 1,\ \alpha_1 = \alpha, \tag{13}$$

where U_i is an index for industry i that is defined over the consumption C_k of a continuum of products k within the industry (as in equation (1) in the paper), and C_k is itself an index that is defined over the consumption $c_k(\omega)$ of a continuum of varieties ω within each product (as in equation (2) in the paper). "Industries" now constitute an upper tier of utility, "products" form an intermediate tier and "varieties" occupy a lower tier. We assume for simplicity that the two industries have the same elasticity of substitution across products within industries (κ) and across varieties within products (σ).

We also assume for simplicity that there are only two countries: home and foreign. The home country is assumed to be skill-abundant relative to the foreign country and industry 1 is assumed to be skill-intensive relative to industry 2. The combination of industry-level variation in factor intensity and country-level variation in factor abundance gives rise to endowment-driven comparative advantage. The skilled wage in the home country is chosen as the numeraire.

To enter an industry i, a firm must incur the sunk entry cost for that industry, which equals $f_{ei}(w_S)^{\beta_i}(w_L)^{1-\beta_i}$, where w_S denotes the skilled wage, w_L corresponds to the unskilled wage and β_i parameterizes industry factor intensity. After the sunk entry cost for an industry is paid, the firm draws its ability and values of product attributes for that industry. The distributions of firm ability and product attributes are independently and identically distributed across industries, so that information about ability within an industry can only be obtained by incurring the sunk entry cost for that industry. The distributions of firm ability and product attributes are also independently and identically distributed across countries, which ensures consistency with the Heckscher-Ohlin model's assumption of common technologies across countries.

The technology for production has the same factor intensity as for entry.[3] While the factor intensity of production varies across industries, all products within an industry are modelled symmetrically and therefore have the same factor intensity.[4] To supply a variety of a product to the domestic market, a firm must incur fixed and variable costs. The variable cost depends upon the firm's ability as in the paper. The fixed and variable costs use the two factors of production with the same proportions. Hence the total cost of serving the domestic market for a firm in industry i is:

$$TC_i = \left(F_{di} + \int_{k \in \Psi_{di}(\varphi_i)} \left[f_{di} + \frac{q_i(\varphi_i, \lambda_{ik})}{\varphi_i} \right] dk \right) (w_S)^{\beta_i} (w_L)^{1-\beta_i}, \quad 1 > \beta_1 > \beta_2 > 0, \tag{14}$$

where $\Psi_{di}(\varphi_i)$ denotes the (endogenous) range of products supplied to the domestic market by a firm with ability φ_i in industry i. The fixed costs of serving an export market and supplying a

[3] Allowing factor intensity differences between entry and production introduces additional interactions with comparative advantage as discussed in Bernard, Redding and Schott (2007).

[4] The symmetry of products within industries is clearly a simplification, but is useful for the law of large numbers results that determine the fraction of products supplied by a firm with a given ability.

product to an export market are modelled analogously.

We determine general equilibrium using an approach very similar to that used in the paper and therefore omit the reporting of similar equations here to conserve space. The measure of products supplied to the domestic and export markets by a firm with a given ability can be determined using expressions analogous to those in the paper. Similarly, the zero-profit and exporting cutoffs for ability can be determined using the same line of reasoning as in the paper. Entry, production and exporting costs all have the same factor intensity, and therefore terms in factor prices cancel from the relevant expressions. There is, however, an important general equilibrium interaction between comparative advantage and firms' product supply decisions. Comparative advantage and trade costs together generate cross-country differences in industry price indices. As in Bernard, Redding and Schott (2007), these differences, in turn, generate greater export opportunities in comparative-advantage industries than comparative-disadvantage industries.

The relative price indices for the two industries vary across countries because of the combination of comparative-advantage-based specialization and trade costs. Specialization leads to a larger mass of domestic firms relative to foreign firms in a country's comparative-advantage industry than in its comparative-disadvantage industry. Variable trade costs introduce a wedge between domestic and export prices for a variety. Additionally, the fixed costs of becoming an exporter imply that not all firms export, and the fixed costs for exporting individual products imply that not all the products supplied domestically are exported. Combining specialization and trade costs, the comparative-advantage industry has a greater mass of lower-price domestic varieties relative to the mass of higher-price foreign varieties than the comparative-disadvantage industry. As a result, the price index in the domestic market is lower relative to the price index in the export market in the comparative-advantage industry. Hence, the degree of competition in the domestic market is higher relative to the degree of competition in the export market in the comparative-advantage industry. These differences in the degree of competition in turn imply that variable profits in the export market are greater relative to variable profits in the domestic market in the comparative-advantage industry than in the comparative-disadvantage industry.

Proposition A *Other things equal, the opening of the closed economy to trade leads to:*

(a) *a greater reduction in the range of products supplied to the domestic market in the comparative-advantage industry than in the comparative-disadvantage industry (*$\Delta\lambda_{d1}^{*H}(\varphi) > \Delta\lambda_{d2}^{*H}(\varphi)$ *and* $\Delta\lambda_{d2}^{*F}(\varphi) > \Delta\lambda_{d1}^{*F}(\varphi)$*),*

(b) *a larger increase in the zero-profit cutoff for ability below which firms exit in the comparative-advantage industry than the comparative-disadvantage industry (*$\Delta\varphi_{1d}^{*H} > \Delta\varphi_{d2}^{*H}$ *and* $\Delta\varphi_{d2}^{*F} >$

$\Delta\varphi_{d1}^{*F}$).

Proof. The relationship between the exporting cutoff ability, φ_{xi}^* and the zero-profit-cutoff ability, φ_{di}^*, in the home country is:

$$\varphi_{xi}^{*H} = \Gamma_i^H \varphi_{di}^{*H}, \qquad \Gamma_i^H \equiv \tau_i \left(\frac{f_{xi}}{f_{di}} \frac{R^H}{R^F}\right)^{\frac{1}{\sigma-1}} \left(\frac{P_i^H}{P_i^F}\right) \left(\frac{\lambda_{di}^*\left(\varphi_{di}^*\right)}{\lambda_{xi}^*\left(\varphi_{xi}^*\right)}\right), \tag{15}$$

where an analogous expression holds for the foreign country. Comparing the free entry conditions in the open and closed economies, the expected value of entry in the open economy is equal to the value for the closed economy plus an additional positive term which captures the expected profits from the export market. Since $\lambda_{xi}^*\left(\varphi_i\right) = \left(\varphi_{xi}^*/\varphi_i\right)\lambda_{xi}^*\left(\varphi_{xi}^*\right)$ and $\varphi_{xi}^* = \Gamma_i \varphi_{di}^*$, this additional positive term is larger, the smaller the value of Γ_i. Dividing equation (15) for the two industries:

$$\frac{\varphi_{x1}^{*H}/\varphi_{d1}^{*H}}{\varphi_{x2}^{*H}/\varphi_{d2}^{*H}} = \frac{\Gamma_1^H}{\Gamma_2^H} = \frac{\tau_1}{\tau_2}\left(\frac{f_{x1}/f_{d1}}{f_{x2}/f_{d2}}\right)^{\frac{1}{\sigma-1}} \left(\frac{\lambda_{d1}^*\left(\varphi_{d1}^*\right)/\lambda_{x1}^*\left(\varphi_{x1}^*\right)}{\lambda_{d2}^*\left(\varphi_{d2}^*\right)/\lambda_{x2}^*\left(\varphi_{x2}^*\right)}\right)\left(\frac{P_1^H/P_2^H}{P_1^F/P_2^F}\right).$$

The remainder of the proof follows the same structure as the proof of Proposition 4 in Bernard, Redding and Schott (2007) and we present an abbreviated version here. The price index for the skill-intensive industry relative to the labor-intensive industry is lower in the skill-abundant country than the labor-abundant country: $P_1^H/P_2^H < P_1^F/P_2^F$. Therefore, in the absence of other differences in parameters across industries except factor intensity (common values of τ_i, F_d, f_d, F_x, f_x, f_e, $g\left(\varphi\right)$ and $z\left(\lambda\right)$ across industries): $\Gamma_1^H < \Gamma_2^H$ and similarly $\Gamma_2^F < \Gamma_1^F$. Hence, the additional positive term in the free entry condition capturing expected profits in the export market is larger in the comparative-advantage industry than the comparative-disadvantage industry.

Noting that $\lambda_{di}^*\left(\varphi\right) = \left(\varphi_{di}^*/\varphi_i\right)\lambda_{di}^*\left(\varphi_{di}^*\right)$, $\lambda_{xi}^*\left(\varphi\right) = \left(\varphi_{xi}^*/\varphi_i\right)\lambda_{xi}^*\left(\varphi_{xi}^*\right)$, and $\varphi_{xi}^{*H} = \Gamma_i^H \varphi_{di}^{*H}$, the expected value of entry in the free entry condition (19) in the paper is monotonically decreasing in φ_{di}^*. Therefore, the comparative-advantage industry's larger increase in the expected value of entry following the opening of trade requires a larger rise in the zero-profit cutoff for firm ability φ_{di}^* in order to restore equality between the expected value of entry and the unchanged sunk entry cost: $\Delta\varphi_{d1}^{*H} > \Delta\varphi_{d2}^{*H}$ and $\Delta\varphi_{d2}^{*F} > \Delta\varphi_{d1}^{*F}$.

Since $\lambda_{di}^*\left(\varphi\right) = \left(\varphi_{di}^*/\varphi_i\right)\lambda_{di}^*\left(\varphi_{di}^*\right)$ and $\lambda_{di}^*\left(\varphi_{di}^*\right)$ is unchanged by the opening of trade, the comparative-advantage industry's larger rise in the zero-profit cutoff for firm ability φ_{di}^* implies a greater reduction in the range of products supplied to the domestic market in the comparative-advantage industry than in the comparative-disadvantage industry: $\Delta\lambda_{d1}^{*H}\left(\varphi\right) > \Delta\lambda_{d2}^{*H}\left(\varphi\right)$ and $\Delta\lambda_{d2}^{*F}\left(\varphi\right) > \Delta\lambda_{d1}^{*F}\left(\varphi\right)$. ■

4. Symmetric Countries

In Section 4.1., we establish the existence and uniqueness of equilibrium with symmetric countries. In Section 4.2., we prove Proposition 1 in the paper, which examines the opening of the closed economy to trade. In Section 4.3., we demonstrate that similar, though more nuanced, predictions hold for reductions in variable trade costs in the open economy equilibrium. In Section 4.4., we prove Proposition 2 in the paper, which examines the relationship between the margins of trade and variable trade costs. In Section 4.5., we prove Proposition 3 in the paper, which is concerned with the relationship between the margins of trade and firm ability. In Section 4.6., we report closed form solutions for symmetric countries with Pareto distributions of firm ability and product attributes.

4.1. Existence and Uniqueness of Equilibrium

The symmetric country general equilibrium is referenced by the sextuple: $\{\varphi_d^*,\ \varphi_x^*,\ \lambda_d^*(\varphi_d^*),\ \lambda_x^*(\varphi_x^*),\ R,\ P\}$. As the model has a recursive structure, the determination of general equilibrium is straightforward. In a first bloc of two equations, $\lambda_d^*(\varphi_d^*)$ and $\lambda_x^*(\varphi_x^*)$ can be determined independently of the other endogenous variables of the model. Using the zero-profit and exporting cutoffs for firm ability and product attributes, the unique equilibrium values of $\lambda_d^*(\varphi_d^*)$ and $\lambda_x^*(\varphi_x^*)$ are implicitly defined by:

$$\int_{\lambda_d^*(\varphi_d^*)}^{\infty}\left[\left(\frac{\lambda}{\lambda_d^*(\varphi_d^*)}\right)^{\sigma-1}-1\right]f_d z(\lambda)\,d\lambda = F_d \tag{16}$$

$$\int_{\lambda_x^*(\varphi_x^*)}^{\infty}\left[\left(\frac{\lambda}{\lambda_x^*(\varphi_x^*)}\right)^{\sigma-1}-1\right]f_x z(\lambda)\,d\lambda = F_x. \tag{17}$$

Having determined $\lambda_d^*(\varphi_d^*)$ and $\lambda_x^*(\varphi_x^*)$ as a function of parameters, a second bloc of two equations determines φ_d^* and φ_x^*. The first of these equations is the relationship between the exporting and zero-profit cutoffs for firm ability, which takes the following form for symmetric countries:

$$\varphi_x^* = \Gamma\varphi_d^*, \qquad\qquad \Gamma \equiv \tau\left(\frac{f_x}{f_d}\right)^{\frac{1}{\sigma-1}}\left(\frac{\lambda_d^*(\varphi_d^*)}{\lambda_x^*(\varphi_x^*)}\right), \tag{18}$$

where Γ has been determined as a function of parameters from the solutions to the first bloc of equations. The second equation is the free entry condition, which with symmetric countries becomes:

$$V = \underbrace{\int_{\varphi_d^*}^{\infty} \left[\int_{\varphi_d^* \lambda_d^*(\varphi_d^*)/\varphi}^{\infty} \left[\left(\frac{\lambda}{\lambda_d^*\left(\varphi_d^*\right)} \frac{\varphi}{\varphi_d^*} \right)^{\sigma-1} - 1 \right] f_d z\left(\lambda\right) d\lambda - F_d \right] g\left(\varphi\right) d\varphi}_{\text{Term } A} \tag{19}$$

$$+ \underbrace{n \int_{\varphi_x^*}^{\infty} \left[\int_{\varphi_x^* \lambda_x^*(\varphi_x^*)/\varphi}^{\infty} \left[\left(\frac{\lambda}{\lambda_x^*\left(\varphi_x^*\right)} \frac{\varphi}{\varphi_x^*} \right)^{\sigma-1} - 1 \right] f_x z\left(\lambda\right) d\lambda - F_x \right] g\left(\varphi\right) d\varphi}_{\text{Term } B} = f_e,$$

where Term A captures expected profits in the domestic market; Term B captures expected profits in the export market; and we have used:

$$\lambda_d^*\left(\varphi\right) = \left(\varphi_d^*/\varphi\right) \lambda_d^*\left(\varphi_d^*\right) \tag{20}$$

$$\lambda_x^*\left(\varphi\right) = \left(\varphi_x^*/\varphi\right) \lambda_x^*\left(\varphi_x^*\right). \tag{21}$$

Using the relationship between the cutoffs for ability (18) to substitute for φ_x^*, the free entry condition (19) can be expressed in terms of a single unknown, φ_d^*:

$$V = \int_{\varphi_d^*}^{\infty} \left[\int_{\varphi_d^* \lambda_d^*(\varphi_d^*)/\varphi}^{\infty} \left[\left(\frac{\lambda}{\lambda_d^*\left(\varphi_d^*\right)} \frac{\varphi}{\varphi_d^*} \right)^{\sigma-1} - 1 \right] f_d z\left(\lambda\right) d\lambda - F_d \right] g\left(\varphi\right) d\varphi \tag{22}$$

$$+ n \int_{\Gamma\varphi_d^*}^{\infty} \left[\int_{\Gamma\varphi_d^* \lambda_x^*(\varphi_x^*)/\varphi}^{\infty} \left[\left(\frac{\lambda}{\lambda_x^*\left(\varphi_x^*\right)} \frac{\varphi}{\Gamma\varphi_d^*} \right)^{\sigma-1} - 1 \right] f_x z\left(\lambda\right) d\lambda - F_x \right] g\left(\varphi\right) d\varphi = f_e,$$

where Γ, $\lambda_d^*\left(\varphi_d^*\right)$ and $\lambda_x^*\left(\varphi_x^*\right)$ have been determined as a function of parameters from the solutions to the first bloc of equations and are invariant with respect to φ_d^*.

By inspection of (22), noting that Γ, $\lambda_d^*\left(\varphi_d^*\right)$ and $\lambda_x^*\left(\varphi_x^*\right)$ have been determined as a function of parameters alone, it is evident that the free entry condition has the following properties: (a) The right-hand side is the constant sunk entry cost: $f_e > 0$; (b) As $\varphi_d^* \to 0$, the left-hand side converges towards infinity: $V \to \infty$; (c) As $\varphi_d^* \to \infty$, the left-hand side converges towards zero: $V \to 0$; (d) The left-hand side is monotonically decreasing in φ_d^*, since a higher value of φ_d^* raises the lower limits of both of the double integrals and also reduces the value of the terms inside the double integrals. It follows that there exists a unique fixed point, where the expected value of entry (V) is equal to the constant sunk entry cost (f_e), as shown in Figure 1.

Having determined the unique equilibrium value of φ_d^*, φ_x^* follows immediately from the relationship between the cutoffs (18) and the solutions to the first bloc of equations. Thus, the first two blocs of equations determine $\{\lambda_d^*\left(\varphi_d^*\right), \lambda_x^*\left(\varphi_x^*\right), \varphi_d^*, \varphi_x^*\}$, from which we can solve for the domestic and exporting cutoffs for product attributes for all firm abilities, $\lambda_d^*\left(\varphi\right)$ and $\lambda_x^*\left(\varphi\right)$ respectively, using (20) and (21).

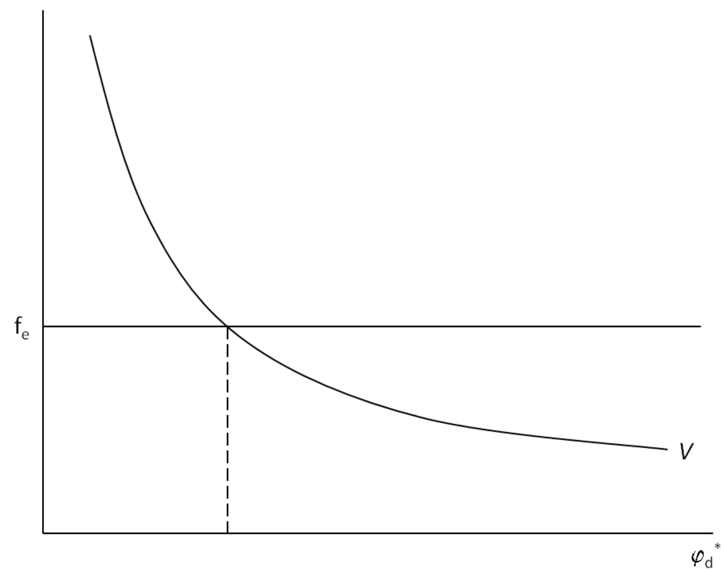

Figure 1: Existence of a Unique Equilibrium φ_d^*

A third bloc of equations determines the remaining components of the equilibrium sextuple: $\{R,\, P\}$. From the free entry condition, $V = [1 - G(\varphi_d^*)]\,\bar{\pi} = f_e$, and the relationship between the mass of firms producing and the mass of entrants, $M = [1 - G(\varphi_d^*)]\,M_e$, total payments to labor used in entry equal total profits:

$$M\bar{\pi} = M_e f_e = L_e,$$

where we have used the choice of numeraire and country symmetry: $w_i = w = 1$. Total payments to labor used in production equal total revenue minus total profits:

$$R - M\bar{\pi} = L_p,$$

from which it follows that total revenue equals the economy's labor endowment and the labor market clears: $R = L$.

To determine the price index for each product, we use the following relationship:

$$P = \left[m_d \left(\frac{1}{\rho \tilde{\varphi}_d} \right)^{1-\sigma} + n m_x \left(\frac{\tau}{\rho \tilde{\varphi}_x} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{23}$$

where we now show that each of the terms on the right-hand side can be written in terms of $\{\varphi_d^*,\, \varphi_x^*,\, \lambda_d^*(\varphi_d^*),\, \lambda_x^*(\varphi_x^*),\, R\}$, which have been already determined above. Weighted average ability in the domestic market is:

$$\tilde{\varphi}_d \equiv \left[\frac{1}{1 - G(\varphi_d^*)} \int_{\varphi_d^*}^{\infty} \left(\varphi \tilde{\lambda}_d(\varphi) \right)^{\sigma - 1} g(\varphi)\, d\varphi \right]^{\frac{1}{\sigma - 1}},$$

$$\tilde{\lambda}_d(\varphi) = \left[\frac{1}{1 - Z(\lambda_d^*(\varphi))}\int_{\lambda_d^*(\varphi)}^{\infty} \lambda^{\sigma-1} z(\lambda)\, d\lambda\right]^{\frac{1}{\sigma-1}},$$

which can be uniquely determined from $\{\varphi_d^*,\ \lambda_d^*(\varphi_d^*)\}$ using (20). Weighted average ability in the export market is:

$$\tilde{\varphi}_x \equiv \left[\frac{1}{1 - G(\varphi_x^*)}\int_{\varphi_x^*}^{\infty} \left(\varphi\tilde{\lambda}_x(\varphi)\right)^{\sigma-1} g(\varphi)\, d\varphi\right]^{\frac{1}{\sigma-1}},$$

$$\tilde{\lambda}_x(\varphi) = \left[\frac{1}{1 - Z(\lambda_x^*(\varphi))}\int_{\lambda_x^*(\varphi)}^{\infty} \lambda^{\sigma-1} z(\lambda)\, d\lambda\right]^{\frac{1}{\sigma-1}},$$

which can be uniquely determined from $\{\varphi_x^*,\ \lambda_x^*(\varphi_x^*)\}$ using (21). The measures of firms supplying each product to the domestic and export markets are:

$$m_d = \left[\int_{\varphi_d^*}^{\infty} \left[1 - Z(\lambda_d^*(\varphi))\right]\left(\frac{g(\varphi)}{1 - G(\varphi_d^*)}\right) d\varphi\right] M, \tag{24}$$

$$m_x = \left[\int_{\varphi_x^*}^{\infty} \left[1 - Z(\lambda_x^*(\varphi))\right]\left(\frac{g(\varphi)}{1 - G(\varphi_x^*)}\right) d\varphi\right] M, \tag{25}$$

where the mass of firms producing is:

$$M = \frac{R}{\bar{r}}, \tag{26}$$

and average firm revenue is:

$$\begin{aligned}\bar{r} \;=\; & \int_{\varphi_d^*}^{\infty}\left[\int_{\lambda_d^*(\varphi)}^{\infty}\left(\frac{\lambda}{\lambda_d^*(\varphi)}\right)^{\sigma-1}\sigma f_d z(\lambda)\, d\lambda\right]\left(\frac{g(\varphi)}{1 - G(\varphi_d^*)}\right) d\varphi \\ & + \left(\frac{1 - G(\varphi_x^*)}{1 - G(\varphi_d^*)}\right)\int_{\varphi_x^*}^{\infty}\left[\int_{\lambda_x^*(\varphi)}^{\infty}\left(\frac{\lambda}{\lambda_x^*(\varphi)}\right)^{\sigma-1}\sigma f_x z(\lambda)\, d\lambda\right]\left(\frac{g(\varphi)}{1 - G(\varphi_x^*)}\right) d\varphi.\end{aligned} \tag{27}$$

From (20) and (21), $\bar{r}$ is uniquely determined by $\{\varphi_d^*,\ \varphi_x^*,\ \lambda_d^*(\varphi_d^*),\ \lambda_x^*(\varphi_x^*)\}$. Given $\bar{r}$ and $R = L$, we immediately obtain M using (26). Given M and $\{\varphi_d^*,\ \varphi_x^*,\ \lambda_d^*(\varphi_d^*),\ \lambda_x^*(\varphi_x^*)\}$, the final two components of the price index, m_d and m_x in (24) and (25), can be determined using (20) and (21). We have therefore determined the price index, which completes the characterization of the equilibrium sextuple $\{\varphi_d^*,\ \varphi_x^*,\ \lambda_d^*(\varphi_d^*),\ \lambda_x^*(\varphi_x^*),\ R,\ P\}$.

4.2. Proof of Proposition 1

Proof. We first examine the impact of the opening of the closed economy to trade on φ_d^*. With symmetric countries, the open economy free entry condition can be written as in (19), which is reproduced below for clarity:

$$V = \underbrace{\int_{\varphi_d^*}^{\infty} \left[\int_{\varphi_d^* \lambda_d^*(\varphi_d^*)/\varphi}^{\infty} \left[\left(\frac{\lambda}{\lambda_d^*(\varphi_d^*)} \frac{\varphi}{\varphi_d^*} \right)^{\sigma-1} - 1 \right] f_d z(\lambda)\, d\lambda - F_d \right] g(\varphi)\, d\varphi}_{\text{Term } A} \tag{28}$$

$$+ \underbrace{n \int_{\varphi_x^*}^{\infty} \left[\int_{\varphi_x^* \lambda_x^*(\varphi_x^*)/\varphi}^{\infty} \left[\left(\frac{\lambda}{\lambda_x^*(\varphi_x^*)} \frac{\varphi}{\varphi_x^*} \right)^{\sigma-1} - 1 \right] f_x z(\lambda)\, d\lambda - F_x \right] g(\varphi)\, d\varphi}_{\text{Term } B} = f_e,$$

where $\lambda_d^*(\varphi_d^*)$ and $\lambda_x^*(\varphi_x^*)$ are determined by parameters alone in (16) and (17) and hence are invariant to $(\varphi_d^*, \varphi_x^*)$. Therefore the free entry condition defines a downward-sloping relationship in $(\varphi_d^*, \varphi_x^*)$ space, as shown in Figure 2.

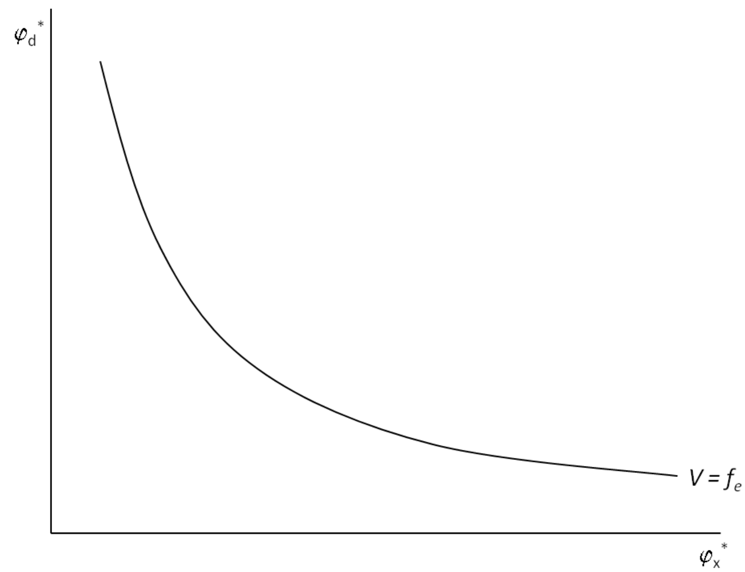

Figure 2: Free Entry Condition

The closed economy free entry condition corresponds to the limiting case of infinitely large trade costs, where $\varphi_x^* \to \infty$ and Term $B \to 0$. As the closed economy is opened to trade and the exporting cutoff for firm ability (φ_x^*) falls to a finite value, Term B becomes strictly positive. It follows that the domestic cutoff for firm ability (φ_d^*) must necessarily rise, in order to reduce Term A and leave the expected value of entry (V) equal to the unchanged sunk entry cost (f_e).

Having established the impact of the opening of trade on φ_d^*, we now turn to the impact on the ranges of products supplied to the domestic and export markets. The domestic product attributes cutoff for each firm ability, $\lambda_d^*(\varphi) = (\varphi_d^*/\varphi)\, \lambda_d^*(\varphi_d^*)$, is monotonically increasing in φ_d^*, while $\lambda_d^*(\varphi_d^*)$

depends solely on parameters. Therefore, as the opening of the closed economy to trade raises φ_d^*, it increases the domestic product attributes cutoff for each firm ability, which implies that all surviving firms drop products with lower values of product attributes from the domestic market.

As the closed economy is opened to trade and φ_x^* falls to a finite value, some surviving firms begin to export. The range of products exported is determined by the export product attributes cutoff for each firm ability, which is given by $\lambda_x^*(\varphi) = (\varphi_x^*/\varphi)\,\lambda_x^*(\varphi_x^*)$, where $\lambda_x^*(\varphi_x^*)$ depends solely on parameters. Given selection into export markets, the new entrants into exporting are high-ability firms, $\varphi_x^* > \varphi_d^*$, and the products added in export markets have high attributes, $\lambda_x^*(\varphi) > \lambda_d^*(\varphi)$.

We now examine the implications of the change in the range of products supplied to each market for firm productivity. As discussed in the paper, firm productivity depends on firm ability (φ) and a revenue-share ($\tilde{r}_{ij}(\varphi,\lambda)$) weighted average of product attributes (λ) across products and markets:

$$\theta_i(\varphi) \equiv \sum_{j=1}^{J} \left[\int_{\lambda_{ij}^*(\varphi)}^{\infty} \theta_{ij}(\varphi,\lambda)\, \tilde{r}_{ij}(\varphi,\lambda)\, d\lambda \right], \tag{29}$$

$$\text{where} \qquad \theta_{ij}(\varphi,\lambda) = \varphi\lambda, \qquad \tilde{r}_{ij}(\varphi,\lambda) \equiv \frac{I_{ij}(\varphi)\, r_{ij}(\varphi,\lambda)\, z(\lambda)}{r_i(\varphi)},$$

and $I_{ij}(\varphi) = 1$ if $\varphi \geq \varphi_{ij}^*$ and a market is served or zero otherwise, and we are concerned here with symmetric countries.

Under autarky, the revenue share of products with attributes $\lambda \in [\lambda_d^{*a}(\varphi), \infty)$ is:

$$\tilde{r}^a(\varphi,\lambda) = \frac{r(\varphi,\lambda)\, z(\lambda)}{r(\varphi)} = \frac{(\varphi\lambda)^{\sigma-1} z(\lambda)}{\int_{\lambda_d^{*a}(\varphi)}^{\infty} (\varphi\lambda)^{\sigma-1} z(\lambda)\, d\lambda}, \qquad \lambda \in [\lambda_d^{*a}(\varphi), \infty), \tag{30}$$

where the superscript a denotes autarky; and the superscript t will be used to denote the open economy below. To characterize the revenue share of products with different attributes in the open economy, consider non-exporters and exporters in turn.

(a) For non-exporters, products with attributes $\lambda \in [\lambda_d^{*a}(\varphi), \lambda_d^{*t}(\varphi))$ are dropped from the domestic market and hence experience a decline in their share of firm revenue. In contrast, products with attributes $\lambda \in [\lambda_d^{*t}(\varphi), \infty)$ experience a rise in their share of firm revenue. Therefore the distribution $\tilde{r}(\varphi,\lambda)$ in the open economy first-order stochastically dominates that in the closed economy, and productivity (29) rises for non-exporters.

(b) For exporters, products with attributes $\lambda \in [\lambda_d^{*a}(\varphi), \lambda_d^{*t}(\varphi))$ are dropped from the domestic market and hence experience a decline in their share of firm revenue. Products with attributes $\lambda \in [\lambda_d^{*t}(\varphi), \lambda_x^{*t}(\varphi))$ experience an ambiguous change in their share of firm revenue:

$$\tilde{r}^t(\varphi,\lambda) = \frac{(\varphi\lambda)^{\sigma-1} z(\lambda)}{\int_{\lambda_d^{*a}(\varphi)}^{\infty} (\varphi\lambda)^{\sigma-1} z(\lambda)\, d\lambda - \Delta_1}, \qquad \lambda \in [\lambda_d^{*t}(\varphi), \lambda_x^{*t}(\varphi)),$$

$$\Delta_1 \equiv \int_{\lambda_d^{*a}(\varphi)}^{\lambda_d^{*t}(\varphi)} (\varphi\lambda)^{\sigma-1} z(\lambda)\, d\lambda - n\tau^{1-\sigma} \int_{\lambda_x^{*t}(\varphi)}^{\infty} (\varphi\lambda)^{\sigma-1} z(\lambda)\, d\lambda.$$

As the sign of Δ_1 is in general ambiguous, $\tilde{r}^t(\varphi,\lambda) \gtreqless \tilde{r}^a(\varphi,\lambda)$ for $\lambda \in [\lambda_d^{*t}(\varphi), \lambda_x^{*t}(\varphi))$. Finally, products with attributes $\lambda \in [\lambda_x^{*t}(\varphi), \infty)$ experience a rise in their share of firm revenue:

$$\begin{aligned}
\tilde{r}^t(\varphi,\lambda) &= \frac{\left[1+n\tau^{1-\sigma}\right](\varphi\lambda)^{\sigma-1} z(\lambda)}{\left[1+n\tau^{1-\sigma}\right]\int_{\lambda_d^{*a}(\varphi)}^{\infty}(\varphi\lambda)^{\sigma-1} z(\lambda)\, d\lambda - \Delta_2}, \quad \lambda \in [\lambda_x^{*t}(\varphi), \infty), \\
\Delta_2 &\equiv \left[\begin{array}{c} \left[1+n\tau^{1-\sigma}\right]\int_{\lambda_d^{*a}(\varphi)}^{\lambda_d^{*t}(\varphi)}(\varphi\lambda)^{\sigma-1} z(\lambda)\, d\lambda \\ +\int_{\lambda_d^{*t}(\varphi)}^{\lambda_x^{*t}(\varphi)} n\tau^{1-\sigma}(\varphi\lambda)^{\sigma-1} z(\lambda)\, d\lambda \end{array}\right] > 0.
\end{aligned}$$

where we have re-written the denominator of $\tilde{r}^t(\varphi,\lambda)$ in a different form. As $\Delta_2 > 0$, $\tilde{r}^t(\varphi,\lambda) > \tilde{r}^a(\varphi,\lambda)$ for $\lambda \in [\lambda_x^{*t}(\varphi), \infty)$.

Irrespective of whether the revenue share of products with intermediate attributes $\lambda \in [\lambda_d^{*a}(\varphi), \lambda_d^{*t}(\varphi))$ rises or falls, the difference between the open economy value of $\tilde{r}(\varphi,\lambda)$ and the closed economy value goes from being negative at low values of $\lambda \in [\lambda_d^{*a}(\varphi), \lambda_d^{*t}(\varphi))$ to being positive at high values of $\lambda \in [\lambda_x^{*t}(\varphi), \infty)$. This is a sufficient condition for the distribution $\tilde{r}(\varphi,\lambda)$ in the open economy to first-order stochastically dominate that in the closed economy. Therefore firm productivity (29) rises for exporters. ■

4.3. Reductions in Variable Trade Costs in the Open Economy Equilibrium

While in the main paper we focus on the impact of opening the closed economy to trade, similar, though more nuanced, predictions hold for reductions in variable trade costs in the open-economy equilibrium.[5]

Proposition B *Reductions in variable trade costs result in within-firm reallocation that leads surviving multiple-product firms to focus on their most successful products:*
(a) *all surviving firms drop products with lower attributes from the domestic market, which raises their productivity,*
(b) *surviving firms that enter the export market drop products with low attributes from the domestic market and add products with higher attributes in the export market, which raises their productivity,*
(c) *surviving exporters drop products with low attributes from the domestic market and add products in the export market. As these products added in the export market have higher attributes than those dropped from the domestic market, but have lower attributes than those previously exported, the effect on their productivity is ambiguous.*

[5]While we concentrate on reductions in variable trade costs, reductions in product fixed exporting costs have similar effects, except where otherwise indicated, as long as there remains selection into export markets: $\lambda_x^*(\varphi) > \lambda_d^*(\varphi)$ and $\varphi_x^* > \varphi_d^*$.

Proof. To establish the proposition, we first characterize $d\varphi_d^*/d\tau$. Using the free entry condition for symmetric countries (19), we define $\Upsilon = V - f_e$. Applying the implicit function theorem, $d\varphi_d^*/d\tau = -\left(d\Upsilon/d\tau\right)/\left(d\Upsilon/d\varphi_d^*\right)$. Substituting for φ_x^*, $\lambda_d^*(\varphi)$ and $\lambda_x^*(\varphi)$ in (19) using (20), (16), (21), (17), and (18), we obtain $dV/d\tau < 0$ and $dV/d\varphi_d^* < 0$. Therefore, we have established that $d\varphi_d^*/d\tau < 0$.

We next characterize $d\varphi_x^*/d\tau$. Differentiating with respect to τ in equation (18), we obtain:

$$\left(\frac{d\varphi_x^*}{d\tau}\frac{\tau}{\varphi_x^*}\right) = 1 + \left(\frac{d\varphi_d^*}{d\tau}\frac{\tau}{\varphi_d^*}\right). \tag{31}$$

It follows that to establish $d\varphi_x^*/d\tau > 0$, it suffices to show that $\left(d\varphi_d^*/d\tau\right)/\left(\tau/\varphi_d^*\right) > -1$. To do so, we again use the implicit function theorem to evaluate $d\varphi_d^*/d\tau = -\left(d\Upsilon/d\tau\right)/\left(d\Upsilon/d\varphi_d^*\right)$. From (20), (16), (21), (17), and (18), we have: $d\lambda_x^*(\varphi)/d\tau = \lambda_x^*(\varphi)/\tau$, $d\lambda_x^*(\varphi)/d\varphi_d^* = \lambda_x^*(\varphi)/\varphi^*$, $d\varphi_x^*/d\tau = \varphi_x^*/\tau$ and $d\varphi_x^*/d\varphi_d^* = \varphi_x^*/\varphi_d^*$. Combining these results with $d\varphi_d^*/d\tau = -\left(d\Upsilon/d\tau\right)/\left(d\Upsilon/d\varphi_d^*\right)$, we obtain $\left(d\varphi_d^*/d\tau\right)/\left(\tau/\varphi_d^*\right) > -1$. Therefore we have established that $d\varphi_x^*/d\tau > 0$.

Since $\lambda_d^*(\varphi) = \left(\varphi_d^*/\varphi\right)\lambda_d^*\left(\varphi_d^*\right)$, where $\lambda_d^*\left(\varphi_d^*\right)$ is invariant to τ, and since $d\varphi_d^*/d\tau < 0$, we have established that $d\lambda_d^*(\varphi)/d\tau < 0$.

Since $\lambda_x^*(\varphi) = \left(\varphi_x^*/\varphi\right)\lambda_x^*\left(\varphi_x^*\right)$, where $\lambda_x^*\left(\varphi_x^*\right)$ is invariant to τ, and since $d\varphi_x^*/d\tau > 0$, we have also established that $d\lambda_x^*(\varphi)/d\tau > 0$.

To determine the impact of the reduction in variable trade costs on firm productivity, denote the values of variables before the reduction by the superscript t and the values of variables after the reduction by the superscript tt. From the above: $\lambda_d^{*tt}(\varphi) > \lambda_d^{*t}(\varphi)$ and $\lambda_x^{*tt}(\varphi) < \lambda_x^{*t}(\varphi)$.

(a) For domestic firms, products with attributes $\lambda \in [\lambda_d^{*t}(\varphi), \lambda_d^{*tt}(\varphi))$ are dropped from the domestic market and hence experience a decline in their share of firm revenue. In contrast, products with attributes $\lambda \in [\lambda_d^{*tt}(\varphi), \infty)$ experience a rise in their share of firm revenue. Therefore the distribution of firm revenue shares $\tilde{r}^{tt}(\varphi, \lambda)$ first-order stochastically dominates the distribution $\tilde{r}^{t}(\varphi, \lambda)$. It follows that the reduction in variable trade costs raises firm productivity for domestic firms. To characterize the magnitude of the rise, note that the share of products in firm revenue for domestic firms before the change in variable trade costs is:

$$\tilde{r}_D^t(\varphi, \lambda) = \frac{(\varphi\lambda)^{\sigma-1} z(\lambda)}{\int_{\lambda_d^{*t}(\varphi)}^{\infty} (\varphi\lambda)^{\sigma-1} z(\lambda)\, d\lambda} \equiv \frac{(\varphi\lambda)^{\sigma-1} z(\lambda)}{AA}, \quad \lambda \in [\lambda_d^{*t}(\varphi), \infty). \tag{32}$$

After the change in variable trade costs, the share of products in firm revenue for domestic firms can be written as:

$$\begin{aligned}\tilde{r}_D^{tt}(\varphi,\lambda) &= \frac{(\varphi\lambda)^{\sigma-1} z(\lambda)}{BB}, \qquad \lambda\in[\lambda_d^{*tt}(\varphi),\infty), \\ BB &\equiv AA-\Delta_3, \\ \Delta_3 &\equiv \int_{\lambda_d^{*t}(\varphi)}^{\lambda_d^{*tt}(\varphi)} (\varphi\lambda)^{\sigma-1} z(\lambda)\, d\lambda > 0.\end{aligned} \tag{33}$$

Therefore, from (32) and (33), the ratio of firm revenue shares after and before the change in variable trade costs for domestic firms is:

$$\frac{\tilde{r}_D^{tt}(\varphi,\lambda)}{\tilde{r}_D^{tt}(\varphi,\lambda)} = \frac{AA}{BB} > 1, \qquad \lambda\in[\lambda_d^{*tt}(\varphi),\infty). \tag{34}$$

(b) For new exporters, the share of products in firm revenue before the reduction in variable trade costs, $\tilde{r}_{NE}^{t}(\varphi,\lambda)$, is the same as for domestic firms in (32) for all $\lambda\in[\lambda_d^{*t}(\varphi),\infty)$. After the reduction in variable trade costs, products with attributes $\lambda\in[\lambda_d^{*t}(\varphi),\lambda_d^{*tt}(\varphi))$ are dropped from the domestic market and therefore experience a decline in their share of firm revenue. On the other hand, products with attributes $\lambda\in[\lambda_d^{*tt}(\varphi),\lambda_x^{*tt}(\varphi))$ experience an ambiguous change in their share of firm revenue:

$$\begin{aligned}\tilde{r}_{NE}^{tt}(\varphi,\lambda) &= \frac{(\varphi\lambda)^{\sigma-1} z(\lambda)}{CC}, \qquad \lambda\in[\lambda_d^{*tt}(\varphi),\lambda_x^{*tt}(\varphi)), \\ CC &\equiv AA-\Delta_4 > BB, \\ \Delta_4 &\equiv \int_{\lambda_d^{*t}(\varphi)}^{\lambda_d^{*tt}(\varphi)} (\varphi\lambda)^{\sigma-1} z(\lambda)\, d\lambda - \int_{\lambda_x^{*tt}(\varphi)}^{\infty} n\left(\tau^{tt}\right)^{1-\sigma} (\varphi\lambda)^{\sigma-1} z(\lambda)\, d\lambda.\end{aligned}$$

Since Δ_4 is ambiguous in sign, we have $\tilde{r}_{NE}^{tt}(\varphi,\lambda) \gtrless \tilde{r}_{NE}^{t}(\varphi,\lambda)$ for $\lambda\in[\lambda_d^{*tt}(\varphi),\lambda_x^{*tt}(\varphi))$. Finally, products with attributes $\lambda\in[\lambda_x^{*tt}(\varphi),\infty)$ experience a rise in their share of firm revenue:

$$\tilde{r}_{NE}^{tt}(\varphi,\lambda) = \frac{\left[1+n\left(\tau^{tt}\right)^{1-\sigma}\right](\varphi\lambda)^{\sigma-1} z(\lambda)}{\left[1+n\left(\tau^{tt}\right)^{1-\sigma}\right] AA-\Delta_5}, \qquad \lambda\in[\lambda_x^{*tt}(\varphi),\infty),$$

$$\Delta_5 \equiv \left[1+n\left(\tau^{tt}\right)^{1-\sigma}\right]\int_{\lambda_d^{*t}(\varphi)}^{\lambda_d^{*tt}(\varphi)} (\varphi\lambda)^{\sigma-1} z(\lambda)\, d\lambda + n\left(\tau^{tt}\right)^{1-\sigma}\int_{\lambda_d^{*tt}(\varphi)}^{\lambda_x^{*tt}(\varphi)} (\varphi\lambda)^{\sigma-1} z(\lambda)\, d\lambda > 0,$$

where we have re-written the denominator of $\tilde{r}_{NE}^{tt}(\varphi,\lambda)$ in a different form. As $\Delta_5>0$, we have $\tilde{r}_{NE}^{tt}(\varphi,\lambda) > \tilde{r}_{NE}^{t}(\varphi,\lambda)$ for $\lambda\in[\lambda_x^{*tt}(\varphi),\infty)$.

Therefore, irrespective of whether the revenue share of products with attributes $\lambda\in[\lambda_d^{*tt}(\varphi),\lambda_x^{*tt}(\varphi))$ rises or falls, the difference between $\tilde{r}^{tt}(\varphi,\lambda)$ and $\tilde{r}^{t}(\varphi,\lambda)$ goes from being negative at low values of λ to being positive at high values of λ. This is a sufficient condition for the distribution $\tilde{r}^{tt}(\varphi,\lambda)$ to first-order stochastically dominate the distribution $\tilde{r}^{t}(\varphi,\lambda)$. It follows that the reduction in variable trade costs raises firm productivity for new exporters. The magnitude of the change in the shares of products in firm revenue for new exporters can be characterized as:

$$
\begin{aligned}
\frac{\tilde{r}_{NE}^{tt}(\varphi,\lambda)}{\tilde{r}_{NE}^{t}(\varphi,\lambda)} &= \frac{AA}{CC} < \frac{AA}{BB}, && \lambda \in [\lambda_d^{*tt}(\varphi), \lambda_x^{*tt}(\varphi)), \\
\frac{\tilde{r}_{NE}^{tt}(\varphi,\lambda)}{\tilde{r}_{NE}^{t}(\varphi,\lambda)} &= \frac{\left[1+n\left(\tau^{tt}\right)^{1-\sigma}\right]AA}{BB+DD} > \frac{AA}{BB} > 1, && \lambda \in [\lambda_x^{*tt}(\varphi), \infty), \\
DD &\equiv n\left(\tau^{tt}\right)^{1-\sigma}\int_{\lambda_x^{*tt}(\varphi)}^{\infty}(\varphi\lambda)^{\sigma-1}z(\lambda)\,d\lambda, \\
DD &< n\left(\tau^{tt}\right)^{1-\sigma}BB.
\end{aligned}
$$

Therefore, from the above expressions and (34), the change in revenue shares of products with low attributes $\lambda \in [\lambda_d^{*t}(\varphi), \lambda_d^{*tt}(\varphi))$ is the same for new exporters as for domestic firms, the change in revenue shares of products with intermediate attributes $\lambda \in [\lambda_d^{*tt}(\varphi), \lambda_x^{*tt}(\varphi))$ is smaller for new exporters than for domestic firms, and the change in revenue shares for products with high attributes $\lambda \in [\lambda_x^{*tt}(\varphi), \infty)$ is larger for new exporters than for domestic firms. These are sufficient conditions for the distribution $\tilde{r}_{NE}^{tt}(\varphi,\lambda)$ to first-order stochastically dominate the distribution $\tilde{r}_{D}^{tt}(\varphi,\lambda)$. It follows that new exporters experience greater productivity growth than domestic firms following the reduction in variable trade costs.

(c) For continuing exporters, products with attributes $\lambda \in [\lambda_d^{*t}(\varphi), \lambda_d^{*tt}(\varphi))$ are dropped from the domestic market and therefore experience a decline in their share of firm revenue. On the other hand, products with attributes $\lambda \in [\lambda_d^{*tt}(\varphi), \lambda_x^{*tt}(\varphi))$ and $\lambda \in [\lambda_x^{*tt}(\varphi), \lambda_x^{*t}(\varphi))$ experience an ambiguous change in their share of firm revenue:

$$
\begin{aligned}
\tilde{r}_{E}^{t}(\varphi,\lambda) &= \frac{(\varphi\lambda)^{\sigma-1}z(\lambda)}{AA+EE}, && \lambda \in [\lambda_d^{*t}(\varphi), \lambda_x^{*t}(\varphi)), \\
EE &\equiv n\left(\tau^{t}\right)^{1-\sigma}\int_{\lambda_x^{*t}(\varphi)}^{\infty}(\varphi\lambda)^{\sigma-1}z(\lambda)\,d\lambda, \\
\tilde{r}_{E}^{tt}(\varphi,\lambda) &= \frac{(\varphi\lambda)^{\sigma-1}z(\lambda)}{AA-FF}, && \lambda \in [\lambda_d^{*tt}(\varphi), \lambda_x^{*tt}(\varphi)), \\
\tilde{r}_{E}^{tt}(\varphi,\lambda) &= \frac{\left[1+n\left(\tau^{tt}\right)^{1-\sigma}\right](\varphi\lambda)^{\sigma-1}z(\lambda)}{AA-FF}, && \lambda \in [\lambda_x^{*tt}(\varphi), \lambda_x^{*t}(\varphi)),
\end{aligned}
$$

$$
FF \equiv \int_{\lambda_d^{*t}(\varphi)}^{\lambda_d^{*tt}(\varphi)}(\varphi\lambda)^{\sigma-1}z(\lambda)\,d\lambda - \left(\left(\frac{\tau^{tt}}{\tau^{t}}\right)^{1-\sigma}EE + n\left(\tau^{tt}\right)^{1-\sigma}\int_{\lambda_x^{*tt}(\varphi)}^{\lambda_x^{*t}(\varphi)}(\varphi\lambda)^{\sigma-1}z(\lambda)\,d\lambda\right)
$$

$$
\frac{1}{AA-FF} \gtrless \frac{1}{AA+EE}, \qquad \text{and} \qquad \frac{1+n\left(\tau^{tt}\right)^{1-\sigma}}{AA-FF} \gtrless \frac{1}{AA+EE}.
$$

Therefore we have $\tilde{r}_{NE}^{tt}(\varphi,\lambda) \gtrless \tilde{r}_{E}^{t}(\varphi,\lambda)$ for $\lambda \in [\lambda_d^{*tt}(\varphi), \lambda_x^{*tt}(\varphi))$ and $\lambda \in [\lambda_x^{*tt}(\varphi), \lambda_x^{*t}(\varphi))$. Finally, products with attributes $\lambda \in [\lambda_x^{*t}(\varphi), \infty)$ also experience an ambiguous change in their share of firm revenue:

$$\tilde{r}_{E}^{t}(\varphi,\lambda)=\frac{\left[1+n\left(\tau^{t}\right)^{1-\sigma}\right](\lambda\varphi)^{\sigma-1}z(\lambda)}{AA+EE},\qquad \lambda\in[\lambda_{x}^{*t}(\varphi),\infty),$$

$$\tilde{r}_{E}^{tt}(\varphi,\lambda)=\frac{\left[1+n\left(\tau^{tt}\right)^{1-\sigma}\right](\lambda\varphi)^{\sigma-1}z(\lambda)}{AA-FF},\qquad \lambda\in[\lambda_{x}^{*t}(\varphi),\infty).$$

As $1/(AA-FF)\gtrless 1/(AA+EE)$, we have $\tilde{r}_{NE}^{tt}(\varphi,\lambda)\gtrless\tilde{r}_{E}^{t}(\varphi,\lambda)$ for $\lambda\in[\lambda_{x}^{*t}(\varphi),\infty)$. To characterize the magnitude of the change in firm revenue shares, consider the ratio of firm revenue shares after and before the reduction in variable trade costs:

$$\frac{\tilde{r}_{E}^{tt}(\varphi,\lambda)}{\tilde{r}_{E}^{t}(\varphi,\lambda)}=\frac{AA+EE}{AA-FF},\qquad \lambda\in[\lambda_{d}^{*tt}(\varphi),\lambda_{x}^{*tt}(\varphi)),$$

$$\frac{\tilde{r}_{E}^{tt}(\varphi,\lambda)}{\tilde{r}_{E}^{t}(\varphi,\lambda)}=\frac{\left[1+n\left(\tau^{tt}\right)^{1-\sigma}\right](AA+EE)}{AA-FF},\qquad \lambda\in[\lambda_{x}^{*tt}(\varphi),\lambda_{x}^{*t}(\varphi)),$$

$$\frac{\tilde{r}_{E}^{tt}(\varphi,\lambda)}{\tilde{r}_{E}^{t}(\varphi,\lambda)}=\frac{\left[1+n\left(\tau^{tt}\right)^{1-\sigma}\right](AA+EE)}{\left[1+n\left(\tau^{t}\right)^{1-\sigma}\right](AA-FF)},\qquad \lambda\in[\lambda_{x}^{*t}(\varphi),\infty).$$

As $1+n\left(\tau^{tt}\right)^{1-\sigma}>1+n\left(\tau^{t}\right)^{1-\sigma}>1$, the change in product firm revenue shares for continuing exporters is smallest for $\lambda\in[\lambda_{d}^{*tt}(\varphi),\lambda_{x}^{*tt}(\varphi))$, largest for $\lambda\in[\lambda_{x}^{*tt}(\varphi),\lambda_{x}^{*t}(\varphi))$, and of intermediate size for $\lambda\in[\lambda_{x}^{*t}(\varphi),\infty)$. With this ordering of changes in firm revenue shares, the change in firm productivity for continuing exporters is in general ambiguous. Additionally, from the above expressions and (34), the change in productivity for continuing exporters can be higher or lower than for domestic firms. ■

4.4. Proof of Proposition 2

Proof. **(a)** With a continuum of symmetric products, the share of products exported to a given market by existing exporters with ability $\varphi\geq\varphi_{x}^{*}$ is $[1-Z(\lambda_{x}^{*}(\varphi))]$. From (21) and (17), $\lambda_{x}^{*}(\varphi)=(\varphi_{x}^{*}/\varphi)\,\lambda_{x}^{*}(\varphi_{x}^{*})$, where $\lambda_{x}^{*}(\varphi_{x}^{*})$ is invariant to τ and Section 4.3. of this appendix established that $d\varphi_{x}^{*}/d\tau>0$. It follows that reductions in variable trade costs reduce φ_{x}^{*} and $\lambda_{x}^{*}(\varphi)$, and hence raise $[1-Z(\lambda_{x}^{*}(\varphi))]$, which establishes part **(a)** of the proposition.

(b) With symmetric countries and common product attributes, the probability that a product is exported to all countries worldwide is $[1-Z(\lambda_{x}^{*}(\varphi))]$. With symmetric countries and country-specific product attributes, the probability that a product is exported to any given country is $[1-Z(\lambda_{x}^{*}(\varphi))]$. In both cases, the expected number of countries to which a product is exported is $[1-Z(\lambda_{x}^{*}(\varphi))]\,n$. From (21) and (17), $\lambda_{x}^{*}(\varphi)=(\varphi_{x}^{*}/\varphi)\,\lambda_{x}^{*}(\varphi_{x}^{*})$, where $\lambda_{x}^{*}(\varphi_{x}^{*})$ is invariant to τ and $d\varphi_{x}^{*}/d\tau>0$. It follows that reductions in variable trade costs reduce φ_{x}^{*} and $\lambda_{x}^{*}(\varphi)$, and hence raise $[1 - Z(\lambda_x^*(\varphi))]$, which establishes part **(b)** of the proposition.

(c) With symmetric countries, the share of firms that export equals $\chi \equiv [1 - G(\varphi_x^*)] / [1 - G(\varphi_d^*)]$. From Section 4.3. of this appendix, $d\varphi_d^*/d\tau < 0$ and $d\varphi_x^*/d\tau > 0$, which implies that $d\chi/d\tau < 0$. It follows that reductions in variable trade costs increase χ, which establishes part **(c)** of the proposition.

(d) With symmetric countries, exports of a firm with a given ability and a given product attribute to a given market can be written as $r_x(\varphi, \lambda) = \tau^{1-\sigma} (\lambda/\lambda_d^*(\varphi_d^*))^{\sigma-1} (\varphi/\varphi_d^*)^{\sigma-1} \sigma f_d$, which is monotonically decreasing in τ, since $\lambda_d^*(\varphi_d^*)$ is invariant to τ and Section 4.3. established that $(d\varphi_d^*/d\tau) / (\tau/\varphi_d^*) > -1$. In contrast, average exports per firm-product-country can be written as:

$$\bar{r}_x(\varphi) = \frac{1}{1 - Z(\lambda_x^*(\varphi))} \int_{\lambda_x^*(\varphi)}^{\infty} \left(\frac{\lambda}{\lambda_x^*(\varphi)} \right)^{\sigma-1} \sigma f_x z(\lambda)\, d\lambda. \tag{35}$$

Since $\lambda_x^*(\varphi) = (\varphi_x^*/\varphi)\, \lambda_x^*(\varphi_x^*)$, where $\lambda_x^*(\varphi_x^*)$ is invariant to τ and $d\varphi_x^*/d\tau > 0$, we have $d\lambda_x^*(\varphi)/d\tau > 0$. Now note that from (35):

$$\frac{d\bar{r}_x(\varphi)}{d\tau} = \left[\underbrace{\frac{dZ(\lambda_x^*(\varphi))/d\lambda_x^*(\varphi)}{1 - Z(\lambda_x^*(\varphi))}}_{>0} \bar{r}_x(\varphi) - \frac{(\sigma-1)}{\lambda_x^*(\varphi)} \bar{r}_x(\varphi) - \frac{\sigma f_x z(\lambda_x^*(\varphi))}{1 - Z(\lambda_x^*(\varphi))} \right] \underbrace{\frac{d\lambda_x^*(\varphi)}{d\tau}}_{>0},$$

which is in general ambiguous in sign depending on the value of $\lambda_x^*(\varphi)$ and the functional form of the cumulative distribution function for product attributes. ■

4.5. Proof of Proposition 3

Proof. **(a)** With a continuum of symmetric products, the share of products exported to a given country by existing exporters with ability $\varphi \geq \varphi_x^*$ is $[1 - Z(\lambda_x^*(\varphi))]$. In this expression, $\lambda_x^*(\varphi) = (\varphi_x^*/\varphi)\, \lambda_x^*(\varphi_x^*)$ is monotonically decreasing in φ and $Z(\lambda)$ is a continuous cumulative distribution function that is increasing in λ. It follows that $[1 - Z(\lambda_x^*(\varphi))]$ is increasing in φ, which establishes part **(a)** of the proposition.

(b) With symmetric countries and common product attributes, the probability that a product is exported to all countries worldwide is $[1 - Z(\lambda_x^*(\varphi))]$. With symmetric countries and country-specific product attributes, the probability that a product is exported to any given country is $[1 - Z(\lambda_x^*(\varphi))]$. In both cases, the expected number of countries to which a product is exported is $[1 - Z(\lambda_x^*(\varphi))]\, n$, where we showed in part **(a)** that $[1 - Z(\lambda_x^*(\varphi))]$ is increasing in φ. We have therefore established part (b) of the proposition.

(c) With symmetric countries, exports of a firm with a given ability and a given product attribute to a given market can be written as $r_x(\varphi, \lambda) = \tau^{1-\sigma} (\lambda/\lambda_d^*(\varphi_d^*))^{\sigma-1} (\varphi/\varphi_d^*)^{\sigma-1} \sigma f_d$, which is monotonically increasing in φ. In contrast, average exports per firm-product-country are (35). Therefore:

$$\frac{d\bar{r}_x(\varphi)}{d\varphi}=\left[\underbrace{\frac{dZ(\lambda_x^*(\varphi))/d\lambda_x^*(\varphi)}{1-Z(\lambda_x^*(\varphi))}}_{>0}\bar{r}_x(\varphi)-\frac{(\sigma-1)}{\lambda_x^*(\varphi)}\bar{r}_x(\varphi)-\frac{\sigma f_x z(\lambda_x^*(\varphi))}{1-Z(\lambda_x^*(\varphi))}\right]\underbrace{\frac{d\lambda_x^*(\varphi)}{d\varphi}}_{<0}$$

which is in general ambiguous in sign depending on the value of $\lambda_x^*(\varphi)$ and the functional form of the cumulative distribution function for product attributes. ■

4.6. Symmetric Country Closed Form Solutions for Pareto Distributions

In this section, we characterize the symmetric country equilibrium for the special case where firm ability is drawn from the Pareto distribution $g(\varphi)=a\varphi_{\min}^a\varphi^{-(a+1)}$ and product attributes are drawn from the Pareto distribution $z(\lambda)=z\lambda_{\min}^z\lambda^{-(z+1)}$. We assume $\varphi_{\min}>0$, $\lambda_{\min}>0$ and $a>z>\sigma>1$, which ensures that firm revenue has a finite mean. Again we choose the wage in one country as the numeraire, which, together with country symmetry, implies that wage in all countries is equal to one.

4.6.1. General Equilibrium with Symmetric Countries

We begin by solving for the equilibrium sextuple $\{\varphi_d^*, \varphi_x^*, \lambda_d^*(\varphi_d^*), \lambda_x^*(\varphi_x^*), P, R\}$. Using the Pareto distribution of product attributes in the zero-profit and exporting cutoff conditions for firm ability (16) and (17), we have:

$$\lambda_d^*(\varphi_d^*)=\left[\frac{\sigma-1}{z-(\sigma-1)}\frac{f_d}{F_d}\right]^{\frac{1}{z}}\lambda_{\min} \tag{36}$$

$$\lambda_x^*(\varphi_x^*)=\left[\frac{\sigma-1}{z-(\sigma-1)}\frac{f_x}{F_x}\right]^{\frac{1}{z}}\lambda_{\min} \tag{37}$$

where we focus on parameter values for which we have an interior equilibrium and selection into export markets: $\lambda_x^*(\varphi_x^*)>\lambda_d^*(\varphi_d^*)>\lambda_{\min}$. Combining (36) and (37) with the relationship between the exporting and zero-profit cutoff firm abilities (18), we obtain:

$$\varphi_x^*=\tau\left(\frac{f_x}{f_d}\right)^{\frac{z-(\sigma-1)}{z(\sigma-1)}}\left(\frac{F_d}{F_x}\right)^{\frac{1}{z}}\varphi_d^* \tag{38}$$

where we again focus on parameter values for which we have an interior equilibrium and selection into export markets: $\varphi_x^*>\varphi_d^*$. Using the Pareto distributions of firm ability and product attributes in the free entry condition (19) yields:

$$v_e=\frac{z}{a-z}\left[F_d\left(\frac{\varphi_{\min}}{\varphi_d^*}\right)^a+nF_x\left(\frac{\varphi_{\min}}{\varphi_x^*}\right)^a\right]=f_e, \tag{39}$$

which together with (38) uniquely determines φ_d^* as a function of parameters alone. Under the assumption of Pareto distributions of firm ability and product attributes, average firm revenue (27) becomes:

$$\bar{r} = \left(\frac{za}{a-z}\right)\left(\frac{\sigma}{\sigma-1}\right)\left[F_d + nF_x\left(\frac{\varphi_d^*}{\varphi_x^*}\right)^a\right],$$

where, from (38), $(\varphi_d^*/\varphi_x^*)$ is a function of parameters alone, which implies that we have determined $\bar{r}$ as a function of parameters alone.

Using the choice of numeraire, the relationship between the mass of firms producing and the mass of entrants ($M = [1 - G(\varphi_d^*)]\, M_e$) and the free entry condition ($V = [1 - G(\varphi_d^*)]\, \bar{\pi} = f_e$), it follows that $L_e = M\bar{\pi}$, as in Section 4.1. above. Since $L_p = R - M\bar{\pi}$, we have $R = L$. The mass of firms producing follows immediately from aggregate revenue and average revenue: $M = R/\bar{r} = L/\bar{r}$, where $\bar{r}$ was determined as a function of parameters above.

Using the Pareto distributions of firm ability and product attributes, the measures of firms supplying each product to the domestic and export markets, (24) and (25), are:

$$m_d = \frac{\left(\frac{a}{a-z}\right)}{\frac{\sigma-1}{z-(\sigma-1)}\frac{f_d}{F_d}}M, \qquad M_x = \frac{\left(\frac{a}{a-z}\right)}{\frac{\sigma-1}{z-(\sigma-1)}\frac{f_x}{F_x}}M$$

where again we focus on an interior equilibrium for which $\frac{a}{a-z} < \frac{\sigma-1}{z-(\sigma-1)}\frac{f_d}{F_d} < \frac{\sigma-1}{z-(\sigma-1)}\frac{f_x}{F_x}$ and M was determined as a function of parameters above.

Finally, weighted-average productivities in the domestic and export markets are:

$$\tilde{\varphi}_d = \left(\frac{z}{z-(\sigma-1)}\right)^{\frac{1}{\sigma-1}}\varphi_d^*\lambda_d^*(\varphi_d^*), \qquad \tilde{\varphi}_x = \left(\frac{z}{z-(\sigma-1)}\right)^{\frac{1}{\sigma-1}}\varphi_x^*\lambda_x^*(\varphi_x^*),$$

where φ_d^*, $\lambda_d^*(\varphi_d^*)$, φ_x^* and $\lambda_x^*(\varphi_x^*)$ were characterized as a function of parameters above.

Having determined the mass of firms producing (M), the measures of firms supplying each product to the domestic and export markets, and weighted average productivity in the domestic and export markets ($\tilde{\varphi}_d$ and $\tilde{\varphi}_x$), the price index for each product, P, follows immediately from (23). This completes the characterization of the equilibrium sextuple $\{\varphi_d^*, \varphi_x^*, \lambda_d^*(\varphi_d^*), \lambda_x^*(\varphi_x^*), P, R\}$.

4.6.2. Margins of Trade with Symmetric Countries

As discussed in the paper, the total exports of a firm with a given ability to a given market can be decomposed into the share of products exported (extensive margin) and average exports per firm-product-country (intensive margin).

With a Pareto distribution of product attributes, the share of products exported to a given country by a firm with ability φ is given by:

$$[1 - Z(\lambda_x^*(\varphi))] = \left(\frac{\lambda_{\min}}{\lambda_x^*(\varphi)}\right)^z = \left(\frac{\lambda_{\min}}{\lambda_x^*(\varphi_x^*)}\frac{\varphi}{\varphi_x^*}\right)^z, \tag{40}$$

where $\lambda_x^*(\varphi_x^*)$ is given by (37); and φ_x^* is determined by (38) and (39). From the above expression, it follows immediately that the extensive margin is monotonically increasing in firm ability. To determine its relationship with variable trade costs, note that from (37) $\lambda_x^*(\varphi_x^*)$ is independent of variable trade costs τ, while from (38) and (39) φ_x^* is increasing in variable trade costs. Therefore reductions in variable trade costs increase the extensive margin.

With a Pareto distribution of product attributes, average exports per firm-product-country are:

$$\begin{aligned}\bar{r}_x(\varphi,\lambda) &= \frac{1}{1 - Z(\lambda_x^*(\varphi))}\int_{\lambda_x^*(\varphi)}^{\infty}\left(\frac{\lambda}{\lambda_x^*(\varphi)}\right)^{\sigma-1}\sigma f_x z(\lambda)\,d\lambda, \\ &= \frac{z}{z-(\sigma-1)}\sigma f_x,\end{aligned} \tag{41}$$

which implies that the intensive margin is *independent* of both variable trade costs and firm ability.

Finally, under the assumption of a Pareto distribution of firm ability, the extensive margin of the share of firms that export is given by:

$$\chi \equiv \frac{[1 - G(\varphi_x^*)]}{[1 - G(\varphi_d^*)]} = \left(\frac{\varphi_d^*}{\varphi_x^*}\right)^a = \frac{1}{\tau}\left(\frac{f_d}{f_x}\right)^{\frac{z-(\sigma-1)}{z(\sigma-1)}}\left(\frac{F_x}{F_d}\right)^{\frac{1}{z}}, \tag{42}$$

which is decreasing in variable trade costs. Hence reductions in variable trade costs increase the share of firms that export.

Note that the extensive margins of the share of products exported and the share of firms that export are also decreasing in product fixed exporting costs, f_x (from (40) and (42) using (37), (38) and (39)). In contrast, the intensive margin of average exports per firm-product-country is increasing in f_x (from (41)).

4.6.3. Pareto Distributions and Heterogeneous Fixed Costs with Symmetric Countries

The result that the intensive margin of average exports per firm-product-country (41) is independent of firm ability requires both a Pareto distribution of product attributes and a product fixed exporting cost that is independent of product attributes. Suppose instead that product fixed exporting costs vary with product attributes: $f_x = \delta_0 \lambda_x^{\delta_1}$. In this case:

$$\bar{r}_x(\varphi,\lambda) = \frac{z}{z-(\sigma-1)}\sigma\delta_0\left(\lambda_x^*(\varphi)\right)^{\delta_1},$$

$$\lambda_x^*(\varphi) = \left(\frac{\varphi_x^*}{\varphi}\right)^{\frac{\sigma-1}{\sigma-1-\delta_1}}\lambda_x^*(\varphi^*),$$

$$\int_{\lambda_x^*(\varphi_x^*)}^{\infty}\left[\left(\frac{\lambda}{\lambda_x^*(\varphi_x^*)}\right)^{\sigma-1}\left(\lambda_x^*(\varphi_x^*)\right)^{\delta_1} - \lambda^{\delta_1}\right]\delta_0 z(\lambda)\,d\lambda = F_x.$$

Therefore, if higher-attribute products have higher fixed exporting costs ($\delta_1 > 0$), the intensive margin is decreasing in firm ability, φ. In contrast, if higher-attribute products have lower fixed exporting costs ($\delta_1 < 0$), the intensive margin is increasing in firm ability, φ. If λ is interpreted as a component of firm productivity that is specific to individual products, $\delta_1 < 0$ is perhaps more natural, since it implies that lower values of λ are associated with higher variable and fixed costs.

5. Asymmetric Countries

In this section, we report the complete derivations for asymmetric countries with Pareto distributions of firm ability and product attributes. In Section 5.1., we characterize the open economy general equilibrium. In Section 5.2., we establish that the core implications of the model in Propositions 1-3 carry over from symmetric to asymmetric countries. In Section 5.3., we derive log linear gravity equations for aggregate trade and the extensive and intensive margins of trade that we estimate in our empirical analysis. In Section 5.4., we show that unobserved variation in firm ability and product attributes has observable implications for the extensive and intensive margins of trade across firms that we examine in our empirical analysis.

5.1. Asymmetric Country Equilibrium

Firm ability is assumed to be Pareto distributed with the probability density function in country i given by $g_i(\varphi) = a\varphi_{\min i}^{a}\varphi^{-(a+1)}$. Countries can differ in terms of their size (labor endowment) and their production technology (as captured by the lower limit of the support of the firm ability distribution, $\varphi_{\min i}$). Product attributes are assumed to be Pareto distributed with the same probability density function across countries: $z(\lambda) = z\lambda_{\min}^{z}\lambda^{-(z+1)}$. To ensure that firm revenue has a finite mean with these Pareto distributions, we assume $a > z > \sigma - 1$. Variable trade costs are allowed to differ across country-partner pairs and need not be symmetric, with domestic trade costless: $\tau_{ij} > 1$ and $\tau_{ii} = 1$. Similarly, product and market fixed costs can differ across country-partner pairs and need not be symmetric: $F_{ij} > 0$ and $f_{ij} > 0$. We choose the wage in one country as the numeraire and, in the presence of asymmetries, wages in general vary across countries.

5.1.1. Product Attributes and Firm Ability Cutoffs

We begin by characterizing the product attributes cutoff for each firm ability ($\lambda_{ij}^{*}(\varphi)$) and the firm ability cutoff (φ_{ij}^{*}) for each source country and destination market.

Equilibrium firm-product profits in country i from serving market j are:

$$\pi_{ij}\left(\varphi, \lambda_{ij}^{*}(\varphi)\right) = \left(\frac{\sigma}{\sigma-1}\tau_{ij}w_i\right)^{1-\sigma}\frac{\left(\varphi\lambda_{ij}^{*}(\varphi)\right)^{\sigma-1}}{\sigma}w_jL_jP_j^{\sigma-1} - w_if_{ij} = 0.$$

Setting equilibrium firm-product profits to zero yields the product attributes cutoff condition for a firm of ability φ:

$$\lambda_{ij}^{*}(\varphi)^{\sigma-1} = \frac{\sigma \frac{w_i}{w_j} f_{ij}}{\left(\frac{\sigma}{\sigma-1}\tau_{ij}w_i\right)^{1-\sigma} L_j P_j^{\sigma-1}\varphi^{\sigma-1}}. \tag{43}$$

Equilibrium firm profits in country i from serving market j are:

$$\begin{aligned}\pi_{ij}\left(\varphi_{ij}^{*}\right) &= \int_{\lambda_{ij}^{*}\left(\varphi_{ij}^{*}\right)}^{\infty}\left(\frac{\sigma}{\sigma-1}\tau_{ij}w_i\right)^{1-\sigma}\frac{\left(\varphi_{ij}^{*}\lambda\right)^{\sigma-1}}{\sigma}w_jL_jP_j^{\sigma-1}z\lambda_{\min}^{z}\lambda^{-(z+1)}d\lambda \\ &\quad -\int_{\lambda_{ij}^{*}(\varphi)}^{\infty} w_i f_{ij} z\lambda_{\min}^{z}\lambda^{-(z+1)}d\lambda - w_iF_{ij}.\end{aligned}$$

Evaluating the integral and using the product attributes cutoff condition (43), equilibrium firm profits in country i from serving market j become:

$$\pi_{ij}(\varphi) = \frac{(\sigma-1)\lambda_{\min}^{z}\sigma^{-\frac{z}{\sigma-1}} f_{ij}^{-\left(\frac{z-(\sigma-1)}{\sigma-1}\right)} L_j^{\frac{z}{\sigma-1}} P_j^{z}}{z-(\sigma-1)}\left(\frac{\sigma w_i\tau_{ij}}{\sigma-1}\right)^{-z}\left(\frac{w_i}{w_j}\right)^{-\frac{z}{\sigma-1}} w_i\varphi^{z} - w_iF_{ij}. \tag{44}$$

Setting equilibrium firm profits to zero yields the firm ability cutoff condition:

$$\left(\varphi_{ij}^{*}\right)^{z} = \frac{\left(\frac{w_i}{w_j}\right)^{\frac{z}{\sigma-1}}\sigma^{\frac{z}{\sigma-1}}F_{ij}f_{ij}^{\left(\frac{z-(\sigma-1)}{\sigma-1}\right)}}{\left(\frac{\sigma-1}{z-(\sigma-1)}\right)\lambda_{\min}^{z}\left(\frac{\sigma}{\sigma-1}w_i\tau_{ij}\right)^{-z}L_j^{\frac{z}{\sigma-1}}P_j^{z}}. \tag{45}$$

Using the product attributes cutoff condition (43) for the lowest-ability firm, $\lambda_{ij}^{*}\left(\varphi_{ij}^{*}\right)$, we obtain the following closed form expression for the product attributes cutoff of the lowest-ability firm:

$$\lambda_{ij}^{*}\left(\varphi_{ij}^{*}\right) = \left[\frac{\sigma-1}{z-(\sigma-1)}\frac{f_{ij}}{F_{ij}}\right]^{\frac{1}{z}}\lambda_{\min}. \tag{46}$$

This expression for the product attributes cutoff of the lowest-ability firm is the same as for symmetric countries and is independent of wages and price indices in each market.

5.1.2. Free Entry

We next use the free entry condition to derive a relationship between the fixed costs of serving each market, the firm ability cutoffs and the sunk entry cost.

Using firm profits (44) and the Pareto distribution of firm ability, and summing across markets, the free entry condition can be written as:

$$\begin{aligned}&\sum_{j=1}^{J}\int_{\varphi_{ij}^{*}}^{\infty}\frac{(\sigma-1)\lambda_{\min}^{z}\sigma^{-\frac{z}{\sigma-1}}f_{ij}^{-\left(\frac{z-(\sigma-1)}{\sigma-1}\right)}}{z-(\sigma-1)}\left(\frac{\sigma w_i\tau_{ij}}{\sigma-1}\right)^{-z}L_j^{\frac{z}{\sigma-1}}P_j^{z}\left(\frac{w_i}{w_j}\right)^{-\frac{z}{\sigma-1}}w_ia\left(\varphi_{ii}^{*}\right)^{a}\varphi^{-(a-z+1)}d\varphi \\ &\qquad -\sum_{j=1}^{J}\int_{\varphi_{ij}^{*}}^{\infty}w_iF_{ij}\left(\varphi_{ii}^{*}\right)^{a}\varphi^{-(a+1)}d\varphi = \frac{w_if_{ei}}{\left(\frac{\varphi_{\min i}}{\varphi_{ii}^{*}}\right)^{a}}.\end{aligned}$$

Evaluating the integral and using the firm ability cutoff condition (45), we obtain:

$$\left(\frac{z}{a-z}\right)\sum_{j=1}^{J}F_{ij}\left(\frac{\varphi_{ii}^{*}}{\varphi_{ij}^{*}}\right)^{a}=\frac{f_{ei}}{\left(\frac{\varphi_{\min i}}{\varphi_{ii}^{*}}\right)^{a}}. \tag{47}$$

5.1.3. Mass of firms

To determine the mass of firms, we use the labor market clearing condition:

$$M_i\sum_{j=1}^{J}\frac{1-G_i\left(\varphi_{ij}^{*}\right)}{1-G_i\left(\varphi_{ii}^{*}\right)}\int_{\varphi_{ij}^{*}}^{\infty}\int_{\lambda_{ij}^{*}(\varphi)}^{\infty}\frac{q_{ij}\left(\varphi,\lambda\right)}{\varphi}z\left(\lambda\right)d\lambda\frac{g_i\left(\varphi\right)}{1-G_i\left(\varphi_{ij}^{*}\right)}d\varphi$$
$$+M_i\sum_{j=1}^{J}\frac{1-G_i\left(\varphi_{ij}^{*}\right)}{1-G_i\left(\varphi_{ii}^{*}\right)}\int_{\varphi_{ij}^{*}}^{\infty}\left[1-Z\left(\lambda_{ij}^{*}\left(\varphi\right)\right)\right]f_{ij}\frac{g_i\left(\varphi\right)}{1-G_i\left(\varphi_{ij}^{*}\right)}d\varphi$$
$$+M_i\sum_{j=1}^{J}\frac{1-G_i\left(\varphi_{ij}^{*}\right)}{1-G_i\left(\varphi_{ii}^{*}\right)}F_{ij}+M_i\frac{f_{ei}}{1-G_i\left(\varphi_{ii}^{*}\right)}=L_i,$$

where total labor demand on the left-hand side equals the labor used by firms for the variable costs of production, plus the labor used for the product and market fixed costs, plus the labor used for the sunk costs of entry.

Using the Pareto distributions of firm ability and product attributes, the labor market clearing condition can be re-written as:

$$M_i\sum_{j=1}^{J}\int_{\varphi_{ij}^{*}}^{\infty}\left[\int_{\lambda_{ij}^{*}(\varphi)}^{\infty}\left(\frac{\sigma}{\sigma-1}\frac{\tau_{ij}w_i}{\varphi}\right)^{-\sigma}\frac{\lambda^{\sigma-1}}{\varphi}w_jL_jP_j^{\sigma-1}\tau_{ij}z\lambda_{\min}^{z}\lambda^{-(z+1)}d\lambda\right]a\left(\varphi_{ii}^{*}\right)^{a}\varphi^{-(a+1)}d\varphi$$
$$+M_i\sum_{j=1}^{J}\int_{\varphi_{ij}^{*}}^{\infty}\left(\frac{\lambda_{\min}}{\lambda_{ij}^{*}\left(\varphi\right)}\right)^{z}f_{ij}a\left(\varphi_{ii}^{*}\right)^{a}\varphi^{-(a+1)}d\varphi$$
$$+M_i\sum_{j=1}^{J}\left(\frac{\varphi_{ii}^{*}}{\varphi_{ij}^{*}}\right)^{a}F_{ij}+M_i\frac{f_{ei}}{\left(\frac{\varphi_{\min i}}{\varphi_{ii}^{*}}\right)^{a}}=L_i.$$

We first evaluate the integral for product attributes and use the product attributes cutoff condition (43). We next evaluate the integral for firm ability and use the firm ability cutoff condition (45). Using the resulting expression and the free entry condition (47), we can solve for the mass of firms:

$$M_i=\frac{\sigma-1}{a\sigma}\frac{L_i}{f_{ei}}\left(\frac{\varphi_{\min i}}{\varphi_{ii}^{*}}\right)^{a}. \tag{48}$$

Hence we obtain the expression for the mass of entrants in the paper:

$$M_{ei}=\frac{M_i}{1-G\left(\varphi_{ii}^{*}\right)}=\frac{\sigma-1}{a\sigma}\frac{L_i}{f_{ei}}, \tag{49}$$

while the mass of firms in each source country i supplying each destination market j is:

$$M_{ij}=\frac{1-G\left(\varphi_{ij}^{*}\right)}{1-G\left(\varphi_{ii}^{*}\right)}M_i=\frac{\sigma-1}{a\sigma}\frac{L_i}{f_{ei}}\left(\frac{\varphi_{\min i}}{\varphi_{ij}^{*}}\right)^{a}. \tag{50}$$

5.1.4. Trade Shares

The aggregate value of trade from source country i to destination market j can be decomposed into the number of exporters and average firm exports conditional on exporting:

$$X_{ij} = \frac{1 - G\left(\varphi_{ij}^*\right)}{1 - G\left(\varphi_{ii}^*\right)} M_i \, \bar{r}_{ij}\left(\varphi\right), \tag{51}$$

where average firm exports conditional on exporting are:

$$\begin{aligned} \bar{r}_{ij}\left(\varphi\right) &= \int_{\varphi_{ij}^*}^{\infty} \left[\left[1 - Z\left(\lambda_{ij}^*\left(\varphi\right)\right)\right] \int_{\lambda_{ij}^*(\varphi)}^{\infty} r_{ij}\left(\varphi, \lambda\right) \frac{z\left(\lambda\right)}{1 - Z\left(\lambda_{ij}^*\left(\varphi\right)\right)} d\lambda \right] \frac{g\left(\varphi\right)}{1 - G\left(\varphi_{ij}^*\right)} d\varphi \\ &= \int_{\varphi_{ij}^*}^{\infty} \left[\int_{\lambda_{ij}^*(\varphi)}^{\infty} \left(\frac{\sigma}{\sigma - 1} w_i \tau_{ij} \right)^{1-\sigma} \left(\varphi\lambda\right)^{\sigma-1} \frac{w_j L_j}{P_j^{1-\sigma}} z \lambda_{\min}^{z} \lambda^{-(z+1)} d\lambda \right] a \left(\varphi_{ij}^*\right)^a \varphi^{-(a+1)} d\varphi. \end{aligned}$$

We first evaluate the integral for product attributes and use the product attributes cutoff condition (43). We next evaluate the integral for firm ability and use the firm ability cutoff condition (45). Together, these steps yield the following expression for average firm exports conditional on exporting:

$$\bar{r}_{ij}\left(\varphi\right) = \frac{\sigma}{\sigma - 1} \frac{az}{a - z} w_i F_{ij}. \tag{52}$$

Combining aggregate trade (51), the mass of firms (48) and average firm exports conditional on exporting (52), and using the Pareto distribution of firm ability to evaluate the probability of exporting, the share of country i in country j's total trade (expenditure) is:

$$\alpha_{ij} = \frac{X_{ij}}{\sum_{k=1}^{J} X_{kj}} = \frac{\left(\varphi_{ij}^*\right)^{-a} \left(L_i / f_{ei}\right) F_{ij} \left(\varphi_{\min i}\right)^a w_i}{\sum_{k=1}^{J} \left(\varphi_{kj}^*\right)^{-a} \left(L_k / f_{ek}\right) F_{kj} \left(\varphi_{\min k}\right)^a w_k}.$$

Using the firm ability cutoff condition (45), we obtain:

$$\alpha_{ij} = \frac{\left(L_i / f_{ei}\right) \left(\varphi_{\min i}\right)^a \tau_{ij}^{-a} w_i^{-\left(\frac{a\sigma - (\sigma-1)}{\sigma - 1}\right)} F_{ij}^{-\left(\frac{a-z}{z}\right)} f_{ij}^{-\frac{a}{z}\left(\frac{z-(\sigma-1)}{\sigma-1}\right)}}{\sum_{k=1}^{J} \left(L_k / f_{ek}\right) \left(\varphi_{\min k}\right)^a \tau_{kj}^{-a} w_k^{-\left(\frac{a\sigma - (\sigma-1)}{\sigma - 1}\right)} F_{kj}^{-\left(\frac{a-z}{z}\right)} f_{kj}^{-\frac{a}{z}\left(\frac{z-(\sigma-1)}{\sigma-1}\right)}}. \tag{53}$$

This expression for the trade share takes a similar form as in the Melitz (2003) model with a Pareto productivity distribution (see Arkolakis et al. 2008 and Chaney 2008), except that in our framework the trade share depends on parameters that determine multi-product firms' choice of the optimal range of products to export. As a result, the trade share depends on product fixed costs (f_{ij}) as well as market fixed costs (F_{ij}), and depends on the dispersion of product attributes within firms ($1/z$) as well as the dispersion of ability across firms ($1/a$).[6]

[6] The exponent on wages reflects the fact that product and market fixed costs are denominated in terms of source-country labor. If the product and market fixed costs were instead denominated in terms of destination-market labor, the exponent on wages would be the same as the exponent on variable trade costs and equal to $-a$.

5.1.5. Wages

We first establish that total payments to labor equal total revenue, before using the equality of total revenue and total expenditure to determine wages. From the free entry condition, total payments to labor used in entry equal total profits:

$$[1 - G(\varphi_{ii}^*)]\,\bar{\pi}_i = w_i f_{ei}, \quad \Rightarrow \quad M_i \bar{\pi}_i = M_{ei} w_i f_{ei} = w_i L_{ei},$$

where we have used $M_i = [1 - G(\varphi_{ii}^*)]\,M_{ei}$. Since total payments to labor used in production equal total revenue minus total profits, it follows immediately that total payments to labor equal total revenue: $R_i = w_i L_i$.

Therefore the equilibrium wage in each country can be determined from the requirement that total revenue equals total expenditure on goods produced there:

$$w_i L_i = \sum_{j=1}^{J} \alpha_{ij} w_j L_j,$$

$$w_i L_i = \sum_{j=1}^{J} \frac{(L_i/f_{ei})\,(\varphi_{\min i})^a\, \tau_{ij}^{-a} w_i^{-\left(\frac{a\sigma-(\sigma-1)}{\sigma-1}\right)} F_{ij}^{-\left(\frac{a-z}{z}\right)} f_{ij}^{-\frac{a}{z}\left(\frac{z-(\sigma-1)}{\sigma-1}\right)}}{\sum_{k=1}^{J} (L_k/f_{ek})\,(\varphi_{\min k})^a\, \tau_{kj}^{-a} w_k^{-\left(\frac{a\sigma-(\sigma-1)}{\sigma-1}\right)} F_{kj}^{-\left(\frac{a-z}{z}\right)} f_{kj}^{-\frac{a}{z}\left(\frac{z-(\sigma-1)}{\sigma-1}\right)}} w_j L_j, \tag{54}$$

which is the expression reported in the paper.

The wage equation (54) provides a system of J equations in the J unknown wages in each country $j = 1, \ldots, J$. Note that this wage equation (54) takes the same form as equation (3.14) on page 1734 of Alvarez and Lucas (2007). Using (54), we can define the following excess demand system:

$$\Xi(\mathbf{w}) = \frac{1}{w_i}\left[\sum_{j=1}^{J} \frac{(L_i/f_{ei})\,(\varphi_{\min i})^a\, \tau_{ij}^{-a} w_i^{-\left(\frac{a\sigma-(\sigma-1)}{\sigma-1}\right)} F_{ij}^{-\left(\frac{a-z}{z}\right)} f_{ij}^{-\frac{a}{z}\left(\frac{z-(\sigma-1)}{\sigma-1}\right)}}{\sum_{k=1}^{J} (L_k/f_{ek})\,(\varphi_{\min k})^a\, \tau_{kj}^{-a} w_k^{-\left(\frac{a\sigma-(\sigma-1)}{\sigma-1}\right)} F_{kj}^{-\left(\frac{a-z}{z}\right)} f_{kj}^{-\frac{a}{z}\left(\frac{z-(\sigma-1)}{\sigma-1}\right)}} w_j L_j - w_i L_i\right],$$

where $\mathbf{w}$ denotes the vector of wages across countries.

Proposition C *There exists a unique wage vector* $\mathbf{w} \in \Re^n_{++}$ *such that* $\Xi(\mathbf{w}) = 0$.

Proof. Note that $\Xi(\mathbf{w})$ has the following properties:

(i) $\Xi(\mathbf{w})$ is continuous

(ii) $\Xi(\mathbf{w})$ is homogeneous of degree zero

(iii) $\mathbf{w} \cdot \Xi(\mathbf{w}) = 0$ for all $\mathbf{w} \in \Re^n_{++}$ (Walras Law)

(iv) There exists a $s > 0$ such that $\Xi_i(\mathbf{w}) > -s$ for each country i and all $\mathbf{w} \in \Re^n_{++}$

(v) If $\mathbf{w}^m \to \mathbf{w}^0$ where $\mathbf{w}^0 \neq 0$ and $w_i^0 = 0$ for some country i, then

$$\max_j \{\Xi_j(\mathbf{w}^m)\} \to \infty.$$

(vi) Gross substitutes

$$\frac{\partial \Xi_i(\mathbf{w})}{\partial w_j} > 0 \quad \text{for all } i,\ j,\ i \neq j, \text{ for all } \mathbf{w} \in \Re^n_{++},$$

$$\frac{\partial \Xi_i(\mathbf{w})}{\partial w_i} < 0 \quad \text{for all } i, \text{ for all } \mathbf{w} \in \Re^n_{++},$$

from which the proposition follows. See Propositions 17.B.2, 17.C.1 and 17.F.3 of Mas-Colell, Whinston and Green (1995) and Theorems 1-3 of Alvarez and Lucas (2007). ■

We have therefore established that the system of equations (54) determines a unique equilibrium wage in each country.

5.1.6. Price Indices

The price index for each product can be written as:

$$P_j^{1-\sigma} = \sum_{i=1}^{J} M_i \frac{1-G\left(\varphi_{ij}^*\right)}{1-G\left(\varphi_{ii}^*\right)} \int_{\varphi_{ij}^*}^{\infty} \left[\left(1-Z\left(\lambda_{ij}^*(\varphi)\right)\right) \int_{\lambda_{ij}^*(\varphi)}^{\infty} \left(\frac{p_{ij}(\varphi,\lambda)}{\lambda}\right)^{1-\sigma} \frac{z(\lambda)}{1-Z\left(\lambda_{ij}^*(\varphi)\right)} d\lambda\right] \frac{g(\varphi)}{1-G\left(\varphi_{ij}^*\right)} d\varphi,$$

which, using the Pareto distributions of product attributes and firm ability, becomes:

$$P_j^{1-\sigma} = \sum_{i=1}^{J} M_i \int_{\varphi_{ij}^*}^{\infty} \left[\int_{\lambda_{ij}^*(\varphi)}^{\infty} \left(\frac{\sigma}{\sigma-1}\frac{\tau_{ij} w_i}{\varphi\lambda}\right)^{1-\sigma} z\lambda_{\min}^{z} \lambda^{-(z+1)} d\lambda\right] a\left(\varphi_{ii}^*\right)^a \varphi^{-(a+1)} d\varphi.$$

We first evaluate the integral for product attributes and use the product attributes cutoff condition (43). We next evaluate the integral for firm ability and use the firm ability cutoff condition (45). Combining the resulting expression with the mass of firms (48), we obtain after some manipulations the expression reported in the paper:

$$P_j^{-a} = \frac{\kappa_P}{w_j^{\left(1-\frac{a}{\sigma-1}\right)} L_j^{\left(1-\frac{a}{\sigma-1}\right)}} \sum_{i=1}^{J} \frac{L_i}{f_{ei}} \left(\varphi_{\min i}\right)^a \tau_{ij}^{-a} w_i^{-\left(a-1+\frac{a}{\sigma-1}\right)} F_{ij}^{-\left(\frac{a-z}{z}\right)} f_{ij}^{-\frac{a}{z}\left(\frac{z-(\sigma-1)}{\sigma-1}\right)}, \tag{55}$$

$$\kappa_P \equiv \lambda_{\min}^{a} \frac{z}{a-z} \left(\frac{z-(\sigma-1)}{\sigma-1}\right)^{-\frac{a}{z}} \sigma^{-\frac{a}{\sigma-1}} \left(\frac{\sigma}{\sigma-1}\right)^{-a}.$$

5.1.7. Welfare

Using the trade share (53), a country's wage can be expressed in terms of its trade share with itself:

$$w_j^a = \frac{1}{\alpha_{ij}} \frac{\left(L_j/f_{ej}\right)\left(\varphi_{\min j}\right)^a w_j^{\left(1-\frac{a}{\sigma-1}\right)} F_{jj}^{-\left(\frac{a-z}{z}\right)} f_{jj}^{-\frac{a}{z}\left(\frac{z-(\sigma-1)}{\sigma-1}\right)}}{\sum_{k=1}^{J}\left(L_k/f_{ek}\right)\left(\varphi_{\min k}\right)^a \tau_{kj}^{-a} w_k^{-\left(a-1+\frac{a}{\sigma-1}\right)} F_{kj}^{-\left(\frac{a-z}{z}\right)} f_{kj}^{-\frac{a}{z}\left(\frac{z-(\sigma-1)}{\sigma-1}\right)}}. \tag{56}$$

Multiplying the wage equation (56) by the price index equation (55), the summation in the denominator of the wage equation cancels with the summation in the price index equation to yield the following equation for welfare:

$$\left(\frac{w_j}{P_j}\right)^a = \frac{1}{\alpha_{ij}} \frac{\lambda_{\min}^a \left(\frac{z}{a-z}\right) L_j^{\frac{a}{\sigma-1}} \varphi_{\min j}^a F_{jj}^{-\left(\frac{a-z}{z}\right)} f_{jj}^{-\frac{a}{z}\left(\frac{z-(\sigma-1)}{\sigma-1}\right)}}{f_{ej} \left(\frac{z-(\sigma-1)}{\sigma-1}\right)^{\frac{a}{z}} \sigma^{\frac{a}{\sigma-1}} \left(\frac{\sigma}{\sigma-1}\right)^a}.$$

In this special case of our model with Pareto distributions for firm ability and product attributes, a country's trade share with itself is a sufficient statistic for the welfare gains from trade, as in Arkolakis, Costinot and Rodriguez-Clare (2010). Even within this special case, our model points to new determinants of the endogenous value of the trade share, which relate to multi-product firms' choice of the optimal range of products to export. Both the trade share and welfare depend on product fixed costs (f_{ij}) as well as market fixed costs (F_{ij}), and depend on the dispersion of product attributes within firms ($1/z$) as well as the dispersion of ability across firms ($1/a$). More generally, if there are departures from Pareto distributions for firm ability and product attributes, a country's trade share with itself, α_{jj}, is no longer a sufficient statistic for the welfare gains from trade in our model.

5.1.8. General Equilibrium

General equilibrium can be referenced by the wage and mass of entrants in each country $\{w_i, M_{ei}\}$. To determine general equilibrium, we use the recursive structure of the model. The system of equations (54) determines a unique equilibrium wage in each country (w_i), while the mass of entrants (M_{ei}) is determined as a function of parameters in (49). Having solved for these two components of the equilibrium vector, all other endogenous variables can be determined. The price index (P_i) follows immediately from wages in (55). The firm ability cutoffs (φ_{ij}^*) for each source country i and destination market j follow immediately from wages and price indices in (45). The product attributes cutoffs ($\lambda_{ij}^*(\varphi)$) for each firm ability φ, source country i and destination market j follow immediately from wages and price indices in (45). The mass of firms producing (M_i) follows immediately from the domestic firm ability cutoff (φ_{ii}^*) and the mass of entrants (M_{ei}) in (48). The mass of firms in each source country i serving each destination market j (M_{ij}) follows immediately from the firm ability cutoffs (φ_{ij}^*) and the mass of entrants (M_{ei}) in (50). The trade share (α_{ij}) follows immediately from wages in (53). This completes the characterization of general equilibrium with asymmetric countries and Pareto distributions.

5.2. Properties of the Asymmetric Country Equilibrium

The asymmetric country equilibrium exhibits similar properties as the symmetric country equilibrium. The free entry condition (47) can be expressed as:

$$\underbrace{\left(\frac{z}{a-z}\right) F_{ii}\left(\frac{\varphi_{\min i}}{\varphi_{ii}^{*}}\right)^{a}}_{\text{Term } A}+\underbrace{\left(\frac{z}{a-z}\right) \sum_{j\neq i} F_{ij}\left(\frac{\varphi_{\min i}}{\varphi_{ij}^{*}}\right)^{a}}_{\text{Term } B}=f_{ei}, \tag{57}$$

where the closed economy corresponds to the limiting case of infinitely large trade costs, for which $\varphi_{ij}^{*} \rightarrow \infty$ for $j \neq i$ and hence Term $B \rightarrow 0$.

As the closed economy is opened to trade, φ_{ij}^{*} falls to a finite value for $j \neq i$ so that Term B becomes strictly positive. It follows that the domestic firm ability cutoff (φ_{ii}^{*}) must necessarily rise following the opening of trade to reduce the value of Term A such that the left-hand side of (57) remains equal to the unchanged sunk entry cost (f_e).

With asymmetric countries, the following relationship between the product attributes cutoffs for firms with different abilities continues to hold:

$$\lambda_{ij}^{*}(\varphi)=\left(\varphi_{ij}^{*}/\varphi\right) \lambda_{ij}^{*}\left(\varphi_{ij}^{*}\right),$$

where the product attributes cutoff of the lowest ability firm in each market ($\lambda_{ij}^{*}\left(\varphi_{ij}^{*}\right)$) in (46) depends solely on parameters and is invariant to φ_{ij}^{*}.

As the domestic firm ability cutoff (φ_{ii}^{*}) rises following the opening of trade, the domestic product attributes cutoff ($\lambda_{ii}^{*}(\varphi)$) also rises. As a result, all firms drop low-attribute products in the domestic market, which reallocates resources towards higher-attribute products and hence raises firm productivity. In addition, high-ability firms begin to export, and hence add products with high attributes in the export market, which further reallocates resources towards higher-attribute products and raises firm productivity.

With asymmetric countries, there is a hierarchy of export markets in terms of their product attributes cutoffs ($\lambda_{ij}^{*}(\varphi)$) in (43) and their firm ability cutoffs (φ_{ij}^{*}) in (45). Both the product attributes cutoff ($\lambda_{ij}^{*}(\varphi)$) in (43) and the firm ability cutoff (φ_{ij}^{*}) in (45) are, other things equal, increasing in variable trade costs. Therefore markets with lower variable trade costs are, other things equal, served by more firms and supplied with more products by each firm. Higher-ability firms have abilities above the firm cutoffs for a larger number of export markets and hence supply more export markets. Additionally, the product attributes cutoff ($\lambda_{ij}^{*}(\varphi)$) in (43) is monotonically decreasing in firm ability, which implies that higher-ability firms also export more products to a given market than lower-ability firms. Finally, firm-product-country exports ($r_{ij}(\varphi, \lambda)$) in equation

(4) in the paper are monotonically increasing in firm ability, which implies that higher-ability firms also export more of a given product with given attributes to a given market.

5.3. *Margins of Trade Across Countries*

Under the assumption of Pareto distributions of firm ability and product attributes, the model yields log linear gravity equations for aggregate trade and the extensive and intensive margins of trade across asymmetric countries.

5.3.1. *Extensive Margin*

We begin with the extensive margin of the measure of firm-product observations with positive trade between countries i and j:

$$O_{ij} = \frac{1 - G\left(\varphi_{ij}^*\right)}{1 - G\left(\varphi_{ii}^*\right)} M_i \int_{\varphi_{ij}^*}^{\infty} \left[1 - Z\left(\lambda_{ij}^*\left(\varphi\right)\right)\right] \frac{g\left(\varphi\right)}{1 - G\left(\varphi_{ij}^*\right)} d\varphi.$$

Using the Pareto distributions of product attributes and firm ability, we obtain:

$$O_{ij} = \left(\frac{\varphi_{ii}^*}{\varphi_{ij}^*}\right)^a M_i \int_{\varphi_{ij}^*}^{\infty} \left(\frac{\lambda_{\min}}{\lambda_{ij}^*\left(\varphi\right)}\right)^z \left(\frac{\varphi_{ij}^*}{\varphi_{\min i}}\right)^a a\left(\varphi_{\min i}\right)^a \varphi^{-(a+1)} d\varphi.$$

Using the following relationship between the product attribute cutoffs for firms with different abilities:

$$\lambda_{ij}^*\left(\varphi\right) = \left(\varphi_{ij}^*/\varphi\right) \lambda_{ij}^*\left(\varphi_{ij}^*\right), \tag{58}$$

and evaluating the integral for firm ability, we obtain:

$$O_{ij} = M_i \left(\frac{\lambda_{\min}}{\lambda_{ij}^*\left(\varphi_{ij}^*\right)}\right)^z \left(\frac{\varphi_{ii}^*}{\varphi_{ij}^*}\right)^a \frac{a}{a - z}.$$

Using the product attribute cutoff for the lowest-ability firm (43) and the mass of firms (48), this becomes:

$$O_{ij} = \left(\frac{\varphi_{\min i}^*}{\varphi_{ij}^*}\right)^a \frac{F_{ij}}{f_{ij}} \frac{L_i}{f_{ei}} \frac{z - (\sigma - 1)}{\sigma - 1} \frac{a}{a - z} \frac{\sigma - 1}{a\sigma}. \tag{59}$$

Using the firm ability cutoff (45), we obtain the following log-linear gravity equation for the extensive margin as a function of source country i characteristics related to country size (e.g. labor endowment and wage), destination market j characteristics related to market size (e.g. labor endowment and price index), bilateral variable and fixed trade costs, and parameters:

$$O_{ij} = \kappa_O \left[w_i^{-\frac{a\sigma}{\sigma-1}} \varphi_{\min i}^a \frac{L_i}{f_{ei}}\right] \left[w_j^{\frac{a}{\sigma-1}} L_j^{\frac{a}{\sigma-1}} P_j^a\right] \left[\tau_{ij}^{-a} F_{ij}^{-\left(\frac{a}{z}-1\right)} f_{ij}^{-\left(1+\frac{a}{\sigma-1}-\frac{a}{z}\right)}\right], \tag{60}$$

$$\kappa_O \equiv \lambda_{\min}^a \left(\frac{z-(\sigma-1)}{\sigma-1} \right)^{-\left(\frac{a}{z}-1\right)} \sigma^{-\frac{a}{\sigma-1}} \left(\frac{\sigma}{\sigma-1} \right)^{-a} \frac{a}{a-z} \frac{\sigma-1}{a\sigma},$$

which is the expression reported in the paper. We estimate this equation using the U.S. as our source country i; observations are destination markets j.

Note that a similar log-linear gravity equation can be derived for the extensive margin of the measure of exporting firms, which using the Pareto distribution of firm ability and the mass of firms (48) can be written as:

$$\frac{1-G\left(\varphi_{ij}^*\right)}{1-G\left(\varphi_{ii}^*\right)} M_i = \left(\frac{\varphi_{\min i}^*}{\varphi_{ij}^*} \right)^a \frac{L_i}{f_{ei}} \frac{\sigma-1}{a\sigma}.$$

Using the firm ability cutoff (45), we obtain the following gravity equation relationship for the measure of exporting firms in terms of source country i characteristics related to country size (e.g. labor endowment and wage), destination market j characteristics related to market size (e.g. labor endowment and price index), bilateral variable and fixed trade costs, and parameters:

$$\frac{1-G\left(\varphi_{ij}^*\right)}{1-G\left(\varphi_{ii}^*\right)} M_i = \left[w_i^{-\frac{a\sigma}{\sigma-1}} \varphi_{\min i}^a \frac{L_i}{f_{ei}} \right] \left[w_j^{\frac{a}{\sigma-1}} L_j^{\frac{a}{\sigma-1}} P_j^a \right] \left[\tau_{ij}^{-a} F_{ij}^{-\frac{a}{z}} f_{ij}^{-\left(\frac{a}{\sigma-1}-\frac{a}{z}\right)} \right] \kappa_E, \tag{61}$$

$$\kappa_E \equiv \lambda_{\min}^a \left(\frac{z-(\sigma-1)}{\sigma-1} \right)^{-\frac{a}{z}} \sigma^{-\frac{a}{\sigma-1}} \left(\frac{\sigma}{\sigma-1} \right)^{-a} \frac{\sigma-1}{a\sigma}.$$

We also estimate this equation using the U.S. as our source country i; observations are destination markets j.

5.3.2. Intensive Margin

We next consider the intensive margin of average firm-product exports between countries i and j conditional on positive trade. To determine this intensive margin, we use aggregate trade (51), the mass of firms (48) and average firm exports conditional on exporting (52), which together yield the following expression for aggregate trade:

$$X_{ij} = \left(\frac{\varphi_{\min i}^*}{\varphi_{ij}^*} \right)^a w_i F_{ij} \frac{L_i}{f_{ei}} \frac{az}{a-z} \frac{\sigma}{\sigma-1} \frac{\sigma-1}{a\sigma}. \tag{62}$$

Using aggregate trade (62) and the measure of firm-product observations with positive trade (59), we obtain the following log-linear gravity equation for the intensive margin of average firm-product exports between countries i and j conditional on positive trade:

$$\frac{X_{ij}}{O_{ij}} = \frac{z\sigma}{z-(\sigma-1)} w_i f_{ij}, \tag{63}$$

which is the expression reported in the paper. This intensive margin (63) is independent of variable trade costs and destination market size. We again estimate this equation using the U.S. as our source country i; observations are destination markets j.

In contrast, the intensive margin of a firm's exports of a product with given attributes to a given market has the following log linear gravity equation representation:

$$r_{ij}(\varphi,\lambda) = \left[w_i^{1-\sigma}\right]\left[w_j L_j P_j^{\sigma-1}\right]\left[\tau_{ij}^{1-\sigma}\right]\left[(\varphi\lambda)^{\sigma-1}\right]\rho^{\sigma-1}, \tag{64}$$

which corresponds to equation (4) in the paper.

Exports of a product with given attributes to a given market (64) again depend on source country characteristics i (wage, w_i), destination market j characteristics (labor endowment, L_j, wage, w_j, and price index, P_j), bilateral variable trade costs (τ_{ij}), characteristics that vary across and within firms (firm ability, φ, and product attributes, λ), and parameters. We estimate this equation using the U.S. as our source country and firm-product-market observations on exports. To the extent that product attributes are common across countries, firm ability and product attributes can be jointly captured by the inclusion of firm-product fixed effects.

5.4. Margins of Trade Across Firms

In the model, differences in exports across firms are driven by the interaction of firm ability and product attributes. While both firm ability and product attributes are unobservable, they have observable implications for firms' extensive and intensive margins that we examine in our empirical work.

5.4.1. Extensive Margin

We begin with a firm's extensive margin of the measure of products exported to a given market:

$$\left[1 - Z\left(\lambda_{ij}^*(\varphi)\right)\right] = \left(\frac{\lambda_{\min}}{\lambda_{ij}^*(\varphi)}\right)^z.$$

Using the relationship between the product attributes cutoffs for firms of different ability (58) and the product attribute cutoff for the lowest-ability firm (46), we obtain:

$$\left[1 - Z\left(\lambda_{ij}^*(\varphi)\right)\right] = \frac{z-(\sigma-1)}{\sigma-1}\frac{F_{ij}}{f_{ij}}\left(\frac{\varphi}{\varphi_{ij}^*}\right)^z, \tag{65}$$

which is the expression reported in the paper. The measure of products exported to a given market (65) is monotonically increasing in a firm's ability φ.

5.4.2. Intensive Margin

A firm's intensive margin of exports of a product with given attributes to a given market is also monotonically increasing its ability, as is immediately evident from:

$$r_{ij}(\varphi,\lambda) = (w_i\tau_{ij})^{1-\sigma} w_j L_j (\rho P_j \varphi\lambda)^{\sigma-1}, \tag{66}$$

which corresponds to equation (4) in the paper.

In contrast, a firm's intensive margin of average exports per product exported to a given market is:

$$\begin{aligned}\bar{r}_{ij}(\varphi,\lambda) &= \int_{\lambda_{ij}^{*}(\varphi)}^{\infty} r_{ij}(\varphi,\lambda)\frac{z(\lambda)}{1-Z\left(\lambda_{ij}^{*}(\varphi)\right)}d\lambda \\ &= \int_{\lambda_{ij}^{*}(\varphi)}^{\infty} r_{ij}(\varphi,\lambda)\, z\lambda_{ij}^{*}(\varphi)^{z}\,\lambda^{-(z+1)}d\lambda, \\ &= \left(\frac{\sigma}{\sigma-1}w_i\tau_{ij}\right)^{1-\sigma}\frac{z\varphi^{\sigma-1}w_jL_jP_j^{\sigma-1}}{z-(\sigma-1)}\lambda_{ij}^{*}(\varphi)^{\sigma-1}.\end{aligned}$$

Using the product attributes cutoff condition (43), average exports per product exported to a given market become:

$$\bar{r}_{ij}(\varphi,\lambda) = \frac{z\sigma}{z-(\sigma-1)}w_if_{ij}, \tag{67}$$

which is the expression reported in the paper. Average exports per product exported to a given market (67) are independent of firm ability.

5.4.3. Observable Implications

Both the number of products exported (65) and average firm-product exports conditional on positive trade (67) are observed in the data. While exports of a product with given attributes are not observed, because product attributes are not observed, the model can be used to derive an observable counterpart. Each firm draws the same distribution of attributes across products within each destination market. Therefore there is a one-to-one relationship in the model between the rank of a product in a firm sales within a given market and the attractiveness of its attributes. Hence the observable counterpart to exports of a product with given attributes is exports of a product of given rank in the firm's exports to a given market (e.g. exports of the firm's largest product).

The model's observable implications for firms' extensive and intensive margins can be therefore summarized as follows. On the one hand, both the number of products exported and exports of a firm's largest product are increasing log linear functions of firm ability. Therefore these two observable variables should be positively related to one another in the data. On the other hand, under the assumption of Pareto distributions, average firm-product exports are unrelated to firm ability in the model. Therefore the number of products exported should have a weaker relationship in the data with average firm-product exports than with exports of the firm's largest product.

6. Empirical Evidence

In Section 6.3., we report additional empirical results on the relationship between largest and average product exports. In Section 6.4., we report additional empirical results on the distribution of exports within firms.

6.1. Trade Liberalization and Product Range

No additional results reported.

6.2. Margins of Trade Across Countries

No additional results reported.

6.3. Margins of Trade Across Firms

As indicated in the main text, Census guidelines prohibit our disclosure of versions of Figure 1 from the main text that display results for firms exporting all numbers of products. Here, we provide additional results that demonstrate the representativeness of our results among all exporting firms. We augment the results from Figure 1 of the main text by reporting the results of firm-level OLS regressions of the log difference between the export value of firms' largest and average products. We use two different independent variables: the number of products exported by the firm and an indicator variable for whether the firm exports more than ten products. We also consider two different samples: exports to all destinations and exports of HS categories 84 and 85 to Canada. As indicated in Table A below, in all four specifications, the gap between largest and average exports is positively and significantly related to the number of products exported. Therefore, consistent with the results reported in the main text and the compositional changes emphasized in the model, exports of a firm's largest product increase more rapidly with the number of products exported than its average exports per product.

6.4. Margins of Trade Within Firms

As indicated in the main text, Census guidelines prohibit our disclosure of versions of Table 4 from the main text that display results for firms exporting all numbers of products. Here, we provide additional results that demonstrate the representativeness of our results among all exporting firms. Table B below augments Table 4 from the main text by reporting the average size distribution for firms' top ten products across all firms exporting more than ten products. As in the main text, we continue to find the distribution of exports to be skewed towards firms' largest products.

Table A: The Gap Between Firms' Largest and Average Product as a Function of the Number of Products Exported

	ln(Difference$_f$)	ln(Difference$_f$)	ln(Difference$_f$)	ln(Difference$_f$)
Number of Products	0.016	0.046		
	0.001	0.001		
i{Number of Products>10}			1.716	1.665
			0.004	0.011
Constant	0.518	0.252	0.417	0.328
	0.002	0.003	0.001	0.002
Observations	186,978	37,191	186,978	37,191
R^2	0.26	0.41	0.51	0.37
Sample				
Products	All	HS 84-85	All	HS 84-86
Destinations	All	Canada	All	Canada

Notes: Table reports results of firm-level OLS regressions. Dependent variable is the log difference between the export value of firms' largest product and their average product. The two independent variables are the number of products exported and an indicator for whether the number of products exported is greater than 10. Robust standard errors are noted below each coeficient Data are for 2002.

6.5. Hierarchies of Markets

No additional results reported.

7. Data Appendix

Data on firm's domestic output, domestic factor use and the number of five-digit SIC goods produced domestically are from the U.S. Census of Manufactures (CMF). Data on export value, ten-digit HS export products and export destinations are from the Linked/Longitudinal Firm Trade Transaction Database (LFTTD).

7.1. Census of Manufactures (CMF) Data

Manufacturing Censuses are conducted every five years and we make use of data from the 1982, 1987, 1992 and 1997 censuses. The sampling unit for each Census is a manufacturing "establishment", or plant, and the sampling frame in each Census year includes information on the mix of products produced by the plant. Very small manufacturing plants (referred to as Administrative Records) are excluded from the analysis unless otherwise noted because data on their mix of products are unavailable. Because product-mix decisions are made at the level of the firm, we aggregate the data to that level for our analysis.[7]

Our definition of "product" is based upon 1987 Standard Industrial Classification (SIC) cat-

[7] Firm identifiers are derived from firms' legal identities, and firms can consist of one or many establishments. Census uses an annual Company Organization Survey both to determine how new firms are organized and to keep track of changes in incumbent firms' ownership structure over time, e.g., the buying and selling of plants, the creation of new plants or the closing of existing plants.

Table B: Distribution of Firm Exports Across Products and Destinations Across Firms that Export More than Ten Productds, 2002

	Rank	All	Products Exported to Canada	HS 84-85 Products Exported to Canada
% of Firm Exports	1	42.2	42.5	41.6
	2	17.0	18.0	18.4
	3	9.8	10.4	10.5
	4	6.5	6.8	6.9
	5	4.7	4.8	4.9
	6	3.5	3.6	3.6
	7	2.8	2.7	2.8
	8	2.2	2.2	2.2
	9	1.8	1.7	1.7
	10	1.5	1.4	1.4
	>10	8.0	5.9	6.0

Notes: Columns report the mean percent of firm exports represented by product with noted rank (from high to low) across firms exporting more than ten, ten-digit HS products in 2002. Second and third columns restrict observations to firms exporting ten products to Canada, and firms exporting ten Machinery and Eletrical products (HS 84-85) to Canada, respectively. Final row reports aggregate share of products with rank greater than 10. Sample sizes across the three columns are 22,239, 5631 and 1807 firms, respectively.

egories, which segment manufacturing output generally according to its end use. We refer to five-digit SIC categories as products or goods.[8] In our sample, aggregate manufacturing contains 1848 products. For each firm in each Census year, we record the set of products in which the firm produces. We also observe firms' total and product-level output. There are an average of 141,561 surviving firms in each Census year for which such extensive-margin adjustments can be observed. For more detail, see Bernard, Redding and Schott (2010).

7.2. Linked/Longitudinal Firm Trade Transaction Database (LFTTD)

This dataset has two components. The first, foreign trade data assembled by the U.S. Census Bureau and the U.S. Customs Bureau, captures all U.S. international trade transactions between 1992 and 2004 inclusive. For each flow of goods across a U.S. border, this dataset records the product classification and the value. Products in the LFTTD are tracked according to ten-digit Harmonized System (HS) categories, which break exported goods into 8572 products. The second component of the LFTTD is the Longitudinal Business Database (LBD) of the U.S. Census Bureau, which records annual employment and survival information for most U.S. establishments.[9] Em-

[8] SIC categories undergo minor revisions in each census year but experienced a major revision in 1987. Census uses an internally generated concordance to map product codes collected in censuses after 1972 to the 1987 revision.

[9] This dataset excludes the U.S. Postal Service and firms in agriculture, forestry and fishing, railroads, education, public administration and several smaller sectors. See Jarmin and Miranda (2002) for an extensive discussion of the

ployment information for each establishment is collected in March of every year and we aggregate the establishment data up to the level of the firm. Matching the annual information in the LBD to the transaction-level trade data yields the LFTTD. We note that our ability to match trade transactions to firms is imperfect: across 1992 to 2004, we match transactions representing 76 and 82 percent of export and import value, respectively. For further details about the construction of the dataset, see Bernard, Jensen and Schott (2009).

7.3. Other Data Sources

Industry-level Canadian tariff data are from Trefler (2004). To match Canadian SIC manufacturing industries to U.S. SIC manufacturing industries we use a concordance developed by Statistics Canada. Using this concordance, we can observe tariff changes for 40 percent (174) of U.S. manufacturing industries representing 50 percent of total manufacturing shipments in 1987. For industries where tariff information is missing, we assign the average of the two-digit SIC industry in which it belongs.

LBD and its construction.

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