

NOTES ON CAPITAL STRUCTURE:  
SECURITY DESIGN: COSTLY STATE  
VERIFICATION AND CONTROL

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## 1 INTRODUCTION

- So far debt and equity were taken as a given and issue was type and extent of agency costs associated with a given security
- Within each theory, relative amount of securities issued (debt-equity ratio) is such that sum of various agency costs is minimized.
- More fundamental question is what determines specific form of the contract that induces investors to supply funds
- Security design papers attempt to derive financial contracts endogenously
- Within security design, the problem may either be
  - allocation of cash flows, or
  - allocation of control rights (or both)

## Optimal Allocation of Cash Flows

- Main idea: *The optimal financial contract between outside investors and penniless entrepreneur who can divert some cash flow, is combination of inside equity and outside debt*
- Common approach: Design an optimal contract for which outsiders are willing to fund project (resolves agency problem)
- Common features:
  - *Entrepreneur can divert part of cash flow*
    - only part of the project's income can be pledged to investors
    - positive NPV projects may not be funded
  - *Investors are passive*
    - entrepreneur's optimal scheme is derived, while investors' claim is on the remaining part of cash flow

## Common conclusion:

- All models aim at deriving specific form of the "left-over" claim held by investor. They structure entrepreneur's incentive problem in such a manner that his claim takes form of inside equity and lender's claim is fixed payment.
  - models predict inside equity and outside debt (not unique)
- Principal-agent theory: Agent's optimal incentive scheme does not generally look like inside equity
  - additional structure must be imposed on agency relationship to generate standard debt contract
- Models differ in their assumption about diversion (verifiability of income):
  - Innes (1990): verifiable
  - Gale and Hellwig (1985): semi-verifiable (at a cost)
  - Bolton and Scharfstein (1990): non-verifiable

## 2 CASH FLOWS VERIFIABLE

- Innes (1990): Given that cash flows are verifiable, there is no scope to literally divert cash flows  
→ interpret diversion in terms of under-provision of effort or extraction of private gains
- Jensen and Meckling (1976) suggest that debt dominates equity when there is an "effort problem" (do not prove optimality of debt)
- Main idea:
  - (i) *The entrepreneur affects the distribution of income by incurring an effort cost*
  - (ii) *A standard debt contract makes the borrower residual claimant for the marginal income above the face value of debt*
  - (iii) *Under certain conditions (MLRP + increasing repayment schedule), this provides the borrower with maximal incentives to exert effort*

## Model:

- Two dates ( $t = 0, 1$ ) no discounting
- Penniless and risk-neutral entrepreneur has following project: at  $t = 0$ , investment  $F$  which generates at  $t = 1$  r.v.  $X \in [0, \bar{X}]$  distributed with d.f.  $q(X)$
- After  $F$  is invested, entrepreneur can exert non-verifiable effort  $e \geq 0$ . Cost of effort  $c(e)$  satisfies  
 $c' > 0$   $c'' > 0$   $c(0) = 0$   $c'(0) = 0$   $\lim_e c(e) = \infty$
- $q(X|e)$  satisfies Monotone Likelihood Ratio Property (MLRP):

$$\frac{\partial}{\partial X} \left( \frac{\frac{\partial q(X|e)}{\partial e}}{q(X|e)} \right) > 0$$

→ higher income is signal of higher effort  
(MLRP implies FOSD)

- Financial contract specifies a repayment  $R(X)$
- Limited liability of entrepreneur and investor:

$$0 \leq R(X) \leq X$$

## The Problem

- Competitive capital markets so investor breaks even  
→ entrepreneur's expected payoff is NPV of project, provided it is funded
- Hence, entrepreneur should exert first-best level of effort, i.e. choose  $e$  such that

$$c'(e) = \int_0^{\bar{X}} X \frac{\partial q(X|e)}{\partial e} dX$$

- But, after outside financing, his incentive to exert effort determined by *retained* return claims → underprovision of effort
- Find the contract  $R(X)$  that maximizes the entrepreneur's utility (return) subject to the incentive compatibility constraint (IC), the lender's participation constraint (PC), and the limited liability constraint (LL) on both sides



$$\max_{\{R(X), e\}} \int_0^{\bar{X}} X - R(X)q(X|e)dX - c(e) - F$$

s.t.

$$(IC) \quad c'(e) = \int_0^{\bar{X}} [X - R(X)] \frac{\partial q(X|e)}{\partial e} dX$$

$$(PC) \quad F = \int_0^{\bar{X}} R(X)q(X|e)dX$$

$$(LL) \quad 0 \leq R(X) \leq X$$

Letting  $\mu$  and  $\lambda$  denote the non-negative Lagrangian multipliers of IC and PC:

$$\begin{aligned} L = & \int_0^{\bar{X}} X - R(X)q(X|e)dX - c(e) \\ & + \mu \left\{ \int_0^{\bar{X}} [X - R(X)] \frac{\partial q(X|e)}{\partial e} dX - c'(e) \right\} \\ & + \lambda \left[ \int_0^{\bar{X}} R(X)q(X|e)dX - F \right] \end{aligned}$$

$$L = \int_0^{\bar{X}} R(X) \left[ \lambda - \mu \frac{\frac{\partial q(X|e)}{\partial e}}{q(X|e)} - 1 \right] q(X|e) dX$$

$$+ \int_0^{\bar{X}} X \left[ 1 + \mu \frac{\frac{\partial q(X|e)}{\partial e}}{q(X|e)} \right] q(X|e) dX - c(e) - \mu c'(e) - \lambda F$$

The problem is therefore linear in  $R(X)$  for all  $X$ , and by inspection the solution is

$$R(X) = \left\{ X \text{ if } \lambda > 1 + \mu \frac{\frac{\partial q(X|e)}{\partial e}}{q(X|e)}; 0 \text{ if } \lambda < 1 + \mu \frac{\frac{\partial q(X|e)}{\partial e}}{q(X|e)} \right.$$

That is, it is optimal to minimize the repayment in those states with the highest ratio  $\left[ \frac{\frac{\partial q(X|e)}{\partial e}}{q(X|e)} \right]$

Assume that the IC-constraint is binding, i.e.  $\mu > 0$ . (Otherwise, effort is first-best effort) Under MLRP,  $\left[ \frac{\frac{\partial q(X|e)}{\partial e}}{q(X|e)} \right]$  is increasing in  $X$ . Hence, there exists a level  $X^*$  such that it is optimal to set

$$R(X) = \{0 \text{ if } X > X^*; X \text{ if } X < X^*\}$$

- Not yet quite a standard debt contract. Add monotonic reimbursement so  $R(X)$  is non decreasing. Without this constraint, entrepreneur could (secretly) borrow, report a high cash flow, reduce his repayment and then repay the loan
- Monotonicity constraint is clearly binding. Optimal repayment schedule subject to this constraint looks like standard debt contract

$$R(X) = \{K \text{ if } X > K; X \text{ if } X \leq K\}$$

where  $K$  is chosen such that

$$\int_0^K X q(X|e) dX + (1 - Q(K|e))K = F$$

and  $e$  is given by IC-constraint

- Investor's LL constraint ( $R(X) \geq 0$ ) has no longer bite, once monotonic repayment constraint is added. For low values of  $X$ , entrepreneur gets nothing and monotonicity constraint precludes that his return grows faster than firm income. Hence, investor must always get at least zero

## Intuition

- Given MLPR, high cash flows are likely to stem from high effort level. Since aim is to elicit high effort level, entrepreneur should be rewarded as much as possible in high states. Consequently repayment should be as high as possible in low states to meet investor's PC-constraint. Additional monotonicity constraint yields standard debt contract.
- Residual claimancy exposes entrepreneur to substantial risk. → risk-neutrality required to avoid trade-off between incentives and insurance
- Full incentive only for incomes about repayment level
- Debt-like contracts have appealing incentives properties when entrepreneur's discretion consists in affecting income level (contrary to discretion over project risk as Jensen and Meckling's asset substitution problem.)

- Standard debt contract no longer optimal if entrepreneur learns information after signing the contract but prior to choosing effort. It offers poor incentives in bad states of nature, as entrepreneur's returns are low (state-contingent debt-overhang problem)
- Remedy (Chiesa (1992)): Make debt contract state-contingent such that repayment in good states is higher than in bad states
- When states are not verifiable, this can be achieved by attaching options (warrants) to acquire equity in firm and a cash/equity settlement option to entrepreneur
- After observing state, but prior to effort choice, investor decides whether to exercise his option. He does so in good states, and entrepreneur can either dilute his equity stake or make cash settlement. He chooses the latter, thereby effectively making high debt payments in good states

### 3 CASH FLOWS SEMI-VERIFIABLE

Townsend (1979)

Gale and Hellwig (1985)

- "Semi-verifiable" refers to perfect but costly state verification (CSV)
- CSV model assume that entrepreneur can hide income rather than consume private benefits or exert low effort levels. Investors, however, can verify income by incurring some costs
- **Main idea:**  
*Optimal contract minimizes expected verification cost. Under certain assumptions, a standard debt contract is optimal*
- There is no need to condition (distribution of) cash flow on an effort choice problem.
- Possibility of hiding income already creates agency problem.

## Model:

- Penniless and risk-neutral entrepreneur has following project:  
Initial investment amount is  $F$  and project generates cash flows  $X \in [0, +\infty)$  with density  $h(X)$
- Sequence of events:
  - loan agreement (fundraising)
  - investment  $F$
  - cash flow  $X$  realized
  - entrepreneur makes a report  $m(X)$  about income
  - investor verifies income or not
  - entrepreneur repays  $R$

## Assumption:

- *Only the entrepreneur observes the cash flow realization  $X$*
- *Investors can perfectly verify cash flow realization at a cost  $c$  (bankruptcy or verification costs)*

## The Problem:

- If verification costs  $c$  are "too large", investors will not verify. With no threat of verification, entrepreneur can report  $X = 0$  and not repay
- Thus, entrepreneur must be induced to repay. Otherwise, project will not be funded
- Suppose  $c$  is sufficiently small to make a verification threat credible. Possible arrangement: verify systematically (i.e. always). This is viable if

$$\int_0^{+\infty} X h(X) dX > F + c$$

- Note that even though verification costs are paid ex-post by investor, *entrepreneur bears these costs ex-ante*, i.e. claim reimburses investor for  $F$  and verification costs  $c$   
 $\Rightarrow$  entrepreneur has incentive to minimize verification costs, while still getting funded



- **Mechanism design:** Principal choose mechanism (i.e. revelation game) that maximizes his expected utility
- Following the realization of  $X$  observed by the entrepreneur:
  - entrepreneur sends message  $m \in M$
  - probability of inspection  $B(m) \in [0, 1]$
  - repayment if cash flow not verified  $R(m)$
  - repayment if cash flow verified  $R(m, X)$
  - The contract specifies  $M, B(m), R(m)$  and  $R(m, X)$  for all  $m \in M$  and  $X \in [0, \infty)$
- **Revelation Principle:**

*For any contract and the actions induced through this contract, there is an incentive-compatible and direct revelation contract that yields the same outcome. In this contract, messages are cash flow reports, i.e.  $M = [0, \infty)$  and entrepreneur reports true cash flow, i.e.  $m(X) = X$*

## Proof

- Consider some initial (indirect) contract  $M, B, R$  (does not need to be incentive compatible)  
Entrepreneur chooses  $m^*(X)$ , i.e.  $m(\cdot)$  such that his payoff is maximized given the contract

Consider a direct contract:  $M' = [0, +\infty)$ ,  $B'$  and  $R'$  with

$$B'(\hat{X}) = B(m^*(\hat{X}))$$

$$R'(\hat{X}) = R(m^*(\hat{X}))$$

$$R'(\hat{X}) = R(m^*(\hat{X}), X)$$

- (1) The two contracts are equivalent, i.e. yield the same outcome as function of  $X$
- (2) Also, entrepreneur truthfully reports  $X$ , i.e. direct contract is incentive-compatible

## Intuition

- (1) Entrepreneur's "computations", i.e. optimal choice of message is incorporated in direct contract (mechanism).
- (2) Entrepreneur would not "lie to himself", i.e. reporting strategy  $m^*(X)$  is optimal  $\forall X$  in initial contract
  - Consider outcome of initial contract where entrepreneur chooses reporting strategy optimally. In particular, assume that for some realizations of  $X$ , optimal strategy involved lying, i.e.  $m(X) \neq X$
  - Now consider direct contract: For all  $X$ , it offers entrepreneur the same terms when revealing  $X$  truthfully, as indirect initial contract with strategy  $m^*(X) \rightarrow$  initial contract may as well be replaced by direct contract. Moreover, this latter contract must be incentive-compatible, otherwise initial contract would not have generated this outcome

## Remarks

- Revelation Principle extends to situations with several agents. This is, it is applicable to any normal form game (static games of incomplete information)
- Revelation principle does not say that the optimal contract is direct truth-telling contract. It is merely a technical device to ease computation of best achievable outcome. It allows to restrict attention to incentive-compatible direct revelation mechanisms
- Revelation Principle does not hold
  - when designer of mechanism (principal) cannot commit to the chosen mechanism. For instance, if he changes rules of the game, after agents reported their types, or if principal cannot commit to long-term contract, i.e. only to one-period contract
  - when agents' actions or messages enter utility functions

Back to the contracting problem:

- Since entrepreneur has to bear (ex-ante) verification costs  $c$ , he selects a contract that provides funding and minimizes expected  $c$
- Formally: investor's PC-constraint will be binding at the optimum. Thus, one can substitute

$$\int_0^{\infty} R(X)h(X)dX = \int_0^{\infty} cB(X)h(X)dX + F$$

into entrepreneur's objective function

$$\int_0^{\infty} [X - R(X)h(X)]dX$$

and minimize expected verification expenditures instead

- **Assumption:** Consider deterministic contracts only:  $\forall \hat{X}, B(\hat{X}) \in \{0, 1\}$ , where  $\hat{X}$  is reported income

- Restate notation:  $\forall \tilde{X} = X, \hat{X}$  and  $\hat{X} \neq X$

$$R(\tilde{X}) = R(\tilde{X}, B(\tilde{X}) = 0)$$

$$R(\tilde{X}, X) = R(\tilde{X}, B(\tilde{X}) = 1)$$

- Thus, optimal contract solves

$$\min c \int_0^\infty B(X)h(X)dX$$

$$(PC) \int_0^\infty B(X)(R(X, X) - c)h(X)dX$$

$$+ \int_0^\infty (1 - B(X))R(X)h(X)dX \geq F$$

$$(IC1) B(X) = 1, B(\hat{X}) = 0 \Rightarrow R(X, X) \leq R(\hat{X})$$

$$(IC2) B(X) = 0, B(\hat{X}) = 0 \Rightarrow R(X) \leq R(\hat{X})$$

$$(IC3) B(X) = 1, B(\hat{X}) = 1 \Rightarrow R(X, X) \leq R(\hat{X}, X)$$

$$(IC4) B(X) = 0, B(\hat{X}) = 1 \Rightarrow R(X) \leq R(\hat{X}, X)$$

$$(LL1) B(X) = 1 \Rightarrow R(X, X) \leq X$$

$$(LL2) B(X) = 0 \Rightarrow R(X) \leq X$$

- Optimal contract is derived in two steps:

(1) Derive restrictions of IC-constraints (truth-telling) on repayment schedule

(2) Design IC-compatible contract that minimizes expected verification costs

## Step (1): IC-compatibility

### Implications of IC3 and IC4:

- In verification states ( $B(\hat{X}) = 1$ ), there are no gains from lying.
- All  $R(\cdot)$  in IC3 and IC4 can be made exclusively dependent on  $X$ , and the equality holds. If there is a penalty for lying strict inequality holds

### Implications of IC2:

- Let  $\bar{R} = \inf\{R(\hat{X}) | B(\hat{X}) = 0\}$  be the lowest repayment for all reports which do not trigger verification.

Consider two possible realizations  $X_1$  and  $X_2$  where  $B(X_1) = 0$  and  $B(X_2) = 0$

If  $X = X_1$ , IC2 requires  $R(X_1) \leq R(\hat{X}_2)$

If  $X = X_2$ , IC2 requires  $R(X_2) \leq R(\hat{X}_1)$

$\Rightarrow R(X_1) = R(X_2) = \bar{R}$

$\Rightarrow$  truthful revelation requires constant repayment in all non-verification states (otherwise, entrepreneur would always report non-verified state with lowest repayment)

## Implications of IC1:

- Suppose  $B(X_1) = 1$  and  $B(X_2) = 0$ . If  $X = X_1$ , IC1 requires that

$$R(X_1, X_1) \leq R(\hat{X}_2) = \bar{R}$$

That is, truthful revelation requires that repayment in verification states is less or equal to  $\bar{R}$ . Otherwise, entrepreneur would report state that does not trigger verification and pay  $\bar{R}$

- $\Rightarrow$  incentive-compatible repayment schedule has following features:
  - No repayment higher than constant repayment  $\bar{R}$  in verification states
  - Repayments in verification states are lower (or equal) than repayment in non-verification states



## Step (2): Min verification costs

- Limited approach taken here: Illustrate how verification expenditures are minimized by standard debt contract

- (i) *Verification and no-verification regions*

$$X_{\bar{R}} \equiv \inf\{X \geq \bar{R} \mid R(X) = \bar{R}, B(X) = 0\}$$

$$B(X) = 1 \text{ for all } X \in [0, X_{\bar{R}})$$

$$B(X) = 0 \text{ for all } X \in [X_{\bar{R}}, \infty)$$

Also, if  $X_{\bar{R}} = \bar{R}$ , the verification region is minimized for a given  $\bar{R}$ . Hence, number of verifications can be reduced by replacing  $R(X, X) \leq \bar{R}$  by  $\bar{R}$  in all states where income is at least  $\bar{R}$

- (ii) *PC-constraint binding*

Suppose entrepreneur has raised more than  $F$ . By reducing the amount of borrowing,  $\bar{R}$  could be lowered and hence some further verification expenditures could be saved  $\rightarrow$  investor's participation constraint is binding (entrepreneur raises  $F$  but no more)

- (iii) *Paying out all for  $B(X) = 1$*   
 If  $R(X, X) < X$ , increasing  $R(X, X)$  to  $X$  relaxes the budget constraint, i.e. allows to lower  $\bar{R} \rightarrow$  optimal repayment scheme looks like a standard debt contract!
- The actual face value of debt  $K = \bar{R}$  is given by

$$\begin{aligned}
 F &= \int_0^K (X - c)h(X)dX + K(1 - H(K)) \\
 &= \int_0^K Xh(X)dX + K(1 - H(K)) - c \int_0^K h(X)dX \\
 &= K - \int_0^K H(X)dX - cH(K)
 \end{aligned}$$

## Conclusion and Comments:

### 1. Summary

- The optimal contract is a standard debt contract, i.e. repayment  $R(X) = \min\{X, K\}$ . The debt contract is optimal in that it minimizes the probability of inspection under the incentive compatibility constraints
- Inefficiency (agency costs of outside financing) take the form of underinvestment when a project's (positive) NPV is smaller than expected verification expenditures

### 2. Bankruptcy and the CSV model:

- Gale and Hellwig (1985) suggest that one interprets verification as bankruptcy: As long as  $\bar{R}$  is paid, i.e. debt serviced, investor does not care about actual income
- Bankruptcy involves costly acquisition of information. Investor or receiver have to assess value of remaining assets. These "verification" costs are paid out in liquidation revenues

- However, CSV model assumes that entrepreneur cannot divert but only *hide* income prior to repayment/verification decision (income must be left in firm in case of verification if he repays less than  $\bar{R}$ ). Thus, to square CSV model with bankruptcy, one needs additional assumptions, e.g.
  - (i) entrepreneur can transform income into perks, but he can only consume it if firm remains solvent, or
  - (ii) following default, lender seizes asset which he values less than the borrower (Lacker (1990))  
 $\Rightarrow$  *Debt is optimal because it minimizes the amount of assets (inefficiently) transferred*

### 3. Verification costs

- Auditing costs, bankruptcy costs, value loss due to seizure of collateral. Secondary market for assets limits the latter.

#### **4. Revelation Principle:**

- Optimality does not require truthful direct mechanism. Many contracts are just as optimal as the standard debt contract, including ones where promised repayments differ from actual ones

#### **5. Risk-Neutrality:**

- With risk-averse entrepreneur (investor risk-neutral), optimal contract also has verification for low incomes and no-verification for high incomes.
- However, risk-aversion implies entrepreneur prefers low repayments when low income (high marginal utility from income)
- I.e. risk-averse entrepreneur gets some insurance for low income levels

## 6. Random Verification:

- With random verification, standard debt is no longer optimal (Mookherjee and Png (1989)). Random verification economizes on verification costs  $\Rightarrow$  contracts with random verification Pareto-dominate deterministic contracts
- Interpretation?  
Maybe, random auditing. But more difficult to interpret the optimal contract as debt. CSV model - when extended - seems closer to a theory of optimal inspection or auditing than to a theory of debt or bankruptcy

## 7. Renegotiation and Commitment:

- Optimal contract is not robust to renegotiation (Gale and Hellwig (1985)). If entrepreneur anticipates renegotiation in verification region, he loses incentives to report truthfully. Dynamic renegotiation involves bargaining under the asymmetric information. First repayment or report by entrepreneur would be signal about true (future) income

## 4 CREDIT TERMINATION THREAT

Bolton and Scharfstein (1990), Bolton and Scharfstein (1996), Hart and Moore (1998), Gromb (LSE 1994)

- **Main idea:** *Entrepreneur makes repayments because of threat of termination, i.e. exclusion from access to further credit*
- Strategic default: Borrower able to repay, but chooses not to do so, e.g., by consuming or diverting project returns before debt becomes due
- *Preventing strategic defaults crucial for profitability of credits, and hence for very existence of credit markets*
- Repetition of lending can discipline borrower even without sufficient collateral or when contracts cannot be enforced (e.g, sovereign debt)
- Borrowers with future projects that need funding may be better off repaying current loan

## Model

- Penniless and risk-neutral entrepreneur has project that requires funding  $F$  and generates cash flow  $X \in \{\underline{X}, \overline{X}\}$ , and where

$$p = Pr(X = \overline{X})$$

$$p\overline{X} + (1 - p)\underline{X} > F$$

$$\underline{X} < F < \overline{X}$$

Note: Contract cannot be written on cash flow (non-verifiable)

- **Result:** Project is not financed since entrepreneur can always report low profit  $\underline{X}$ . Given  $\underline{X} < F$ , the pledgeable part of the project is less than the investment outlay, and investor does not break even.
- Cash flow has verifiable component  $\underline{X}$ , and a non-verifiable component  $\overline{X} - \underline{X}$
- The no-investment result relies also on limited liability of entrepreneur, and that project/firm has no other assets than payoff  $X$



## Two-period Model

- Suppose the risk-neutral and penniless entrepreneur has a sequence of two identical projects, as described above. Within each period, entrepreneur decides to invest  $F$  or not.
- For simplicity, suppose cash flows across periods are independent, and no discounting
- **Result:** *Projects cannot be financed independently, even though they are independent*
- If investor can only finance one isolated project, he is in the same position as investor in one-period model  
⇒ it is only possible to depart from the non-funding outcome if contract links financing of two projects

## Optimal contract under full commitment

- Suppose that once contract is signed, it is binding and cannot be altered.
- Suppose also that investor has full bargaining power initially. From discussion of one-period model it follows that repayment in second-period cannot exceed  $\underline{X}$ . Since the game ends at that time, there is no (last-period) termination threat
- However, investor can now condition funding of second project on repayment made after first project. Investor can threaten to cut off funding if investor does not make a sufficiently high repayment
- This threat is credible since  $\overline{X} - \underline{X} < F$ . That is, by assumption, investor is needed for second project
- Entrepreneur is induced to repay more (if feasible), since second project has positive NPV

**Result:** *Given  $\overline{X} - \underline{X} < F$  (investor needed for second project), investor funds first project if*

$$p^2(\overline{X} - \underline{X}) - (1 + p)(F - \underline{X}) > 0$$

*At the end of period 1, entrepreneur repays either  $\underline{R} = \underline{X}$  and second project is not funded, or  $R = p(\overline{X} - \underline{X}) + \underline{X}$  and second project is funded*

- Optimal contract may be interpreted as **debt**: Entrepreneur borrows  $F$  against promised repayment  $\overline{R}$ . If entrepreneur fails to repay  $\overline{R}$ , investor shuts down further operations
- **Proof** of above result:  
 Repayment in second-period cannot exceed  $\underline{X}$ , irrespective of actual outcome due to non-verifiability of  $(\overline{X} - \underline{X})$ . As a result, investor cannot break even on second project. Nonetheless, if he can extract sufficient repayment from first-period project, he will be willing to fund second-period project.

Investor has all bargaining power and extracts all surplus. Entrepreneur is just indifferent between repaying more and investing, or skipping second period-project (entrepreneur's incentive-compatibility constraint binds):

$$\begin{aligned}
 (\bar{X} - \underline{X}) + [p(\bar{X} - \underline{X}) + (\underline{X} - \underline{R})] - (\bar{R} - \underline{R}) &= (\bar{X} - \underline{X}) \\
 \rightarrow \bar{R} &= p(\bar{X} - \underline{X}) + \underline{R}
 \end{aligned}$$

Investor's profit: In first project, he loses  $F - \underline{X}$ . In second project he gains  $\bar{R} - F$ . Total return:

$$\begin{aligned}
 &-(F - \underline{X}) + p(\bar{R} - F) \\
 &= -(F - \underline{X}) + p[p(\bar{X} - \underline{X}) + \underline{X} - F] \\
 &= p^2(\bar{X} - \underline{X}) - (1 + p)(F - \underline{X})
 \end{aligned}$$

This is positive (so investor will finance first project) iff

$$p(\bar{X} - \underline{X}) > \left[\frac{1}{p} + 1\right](F - \underline{X})$$

Entrepreneur's payoff:

$$\begin{aligned}
 (\underline{X} - \underline{R}) + p(\bar{X} - \underline{X}) - (\bar{R} - \underline{R}) + p(\bar{X} - \underline{X}) + (\underline{X} - \underline{R}) \\
 = p(\bar{X} - \underline{X})
 \end{aligned}$$

- There are two reasons why investor cuts off financing after low repayment  $\underline{R}$ :
  - avoids losing money  $F - \underline{X}$  in second-period
  - prevents strategic defaults, i.e. induces entrepreneur to repay  $\overline{R}$  if high income realized in first project
- However, optimal contract leads to ex-post inefficiencies: Firm is liquidated, i.e. no second project, if first project did not succeed ( $X = \underline{X}$ ), even though second project has positive NPV.
- **Renegotiation?** Is optimal (full commitment) contract renegotiation-proof? Yes:
  - if entrepreneur repays, he wants the project to go ahead (efficient outcome)
  - if he defaults, the investor does not want to finance (because he loses in single-project model)

- Renegotiation-proofness relies on assumption that investor has all bargaining power, i.e. extracts all surplus from second-project
- Assume instead that investor does not have full bargaining power and only gets fraction  $(1 - \gamma)$  of surplus generated by the second project, i.e.

$$(1 - \gamma)[p(\overline{X} - \underline{X}) + (\underline{X} - F)]$$

- Since entrepreneur can now extract  $\gamma$  of surplus, he has incentive to deviate from optimal (full commitment) contract. By repaying  $\underline{R}$  and renegotiate, he can secure himself more than  $(\overline{X} - \underline{X})$   
 $\Rightarrow$  contract is no longer renegotiation proof.
- Entrepreneur's incentive-compatibility constraint:

$$\begin{aligned} & (\overline{X} - \underline{X}) + [p(\overline{X} - \underline{X}) + (\underline{X} - \underline{R})] - (\overline{R}' - \underline{R}) \\ & = (\overline{X} - \underline{X}) + \gamma[p(\overline{X} - \underline{X}) + (\underline{X} - F)] \\ \rightarrow & p(\overline{X} - \underline{X}) - (\overline{R}' - \underline{R}) = \gamma[p(\overline{X} - \underline{X}) + (\underline{X} - F)] \\ \rightarrow & \overline{R}' = (1 - \gamma)[p(\overline{X} - \underline{X}) + \underline{X}] + \gamma F \end{aligned}$$

- Renegotiation-proof contract now contains repayment  $\bar{R}'$ . As a result, investor's returns are lower
  - In first project, he still loses  $F - \underline{X}$
  - In second project, he merely gains  $\bar{R}' - F$

Total return is

$$\begin{aligned}
 & -(F - \underline{X}) + p(\bar{R}' - F) \\
 = & -(F - \underline{X}) + p[(1 - \gamma)[p(\bar{X} - \underline{X}) + \underline{X}] + \gamma F - F] \\
 = & -[1 + p(1 - \gamma)](F - \underline{X}) + p^2(1 - \gamma)(\bar{X} - \underline{X})
 \end{aligned}$$

- Investor provides funding initially, iff

$$p(\bar{X} - \underline{X}) > \left[ \frac{1}{p(1 - \gamma)} + 1 \right] (F - \underline{X})$$

Hence, to get finance, i.e. investor breaking even, it may be necessary to increase the investor's bargaining position

## Case where Entrepreneur has Bargaining Power Initially

- Maximize entrepreneur's utility subject to investor just breaking even.
- Let  $\beta(R)$  denote the probability of refinancing following a repayment  $R$  (interpretation: partial liquidation). Previously, optimal contract was

$$\beta(\overline{R}) = 1 \text{ and } \beta(\underline{R}) = 0$$

Now aim is to minimize inefficiency, i.e. incidence of no funding of second positive NPV project, subject to investor breaking even

- Note that it is optimal to set  $\underline{R} = \underline{X}$ . For  $\underline{R} < \underline{X}$ , investor would have to liquidate firm more often to realize given return. This would imply more inefficiencies, i.e. less second-period projects



- Hence, incentive-compatible repayment, given  $X = \bar{X}$  in first project is:

$$\begin{aligned}
& (\bar{X} - \underline{X}) + \beta(\bar{R}'') [p(\bar{X} - \underline{X}) + (\underline{X} - \underline{R})] - \underline{R}'' \\
&= (\bar{X} - \underline{X}) + \beta(\underline{R}) [p(\bar{X} - \underline{X}) + (\underline{X} - \underline{R})] - \underline{R} \\
&\rightarrow \beta(\bar{R}'') p(\bar{X} - \underline{X}) - \bar{R}'' = \beta(\underline{R}) p(\underline{X} - \underline{X}) \\
&\rightarrow \bar{R}'' = [\beta(\bar{R}'') - \beta(\underline{R})] p(\bar{X} - \underline{X}) + \underline{R}
\end{aligned}$$

Investor's (IR)

$$\begin{aligned}
0 &= -(F - \underline{X}) - (1 - p)\beta(\underline{R})[F - \underline{X}] \\
&+ p\beta(\bar{R}'') [[\beta(\bar{R}'') - \beta(\underline{R})] p(\bar{X} - \underline{X}) - (F - \underline{X})]
\end{aligned}$$

Hence,  $\beta(\bar{R}'') = 1$  and  $\beta(\underline{R}) = 0$  solves (IR)

## Comments:

- *Lending = Giving + Selling* (Gromb LSE94)
- In first period, investor sinks  $F$ , which yields him a loss  $F - \underline{X}$  (*development phase*)
- Provided first period project has been successful, investor sells second investment  $F$  to entrepreneur at a price equal to the expected profit (*surplus extraction phase*)  
→ Bolton-Scharfstein model tells development story.
- After first cash flow is realized, entrepreneur has some wealth. (Cash flows  $(\bar{X} - \underline{X})$  are his because they are not verifiable). Now, it may be possible to finance both projects.
- However, to generate outcome where funding occurs due to repeated lending Bolton-Scharfstein need yet another **assumption**:  
*Investor is indispensable for realization of second project*

- This assumption implies that investor's participation, or at least his agreement, is necessary for second project.
- Otherwise, entrepreneur can self-finance ( $\overline{X} - \underline{X} > F$ ) and investor never breaks even. Or, entrepreneur raises finance for second project from a (possibly the same) investor ( $(\overline{X} - \underline{X}) + \underline{X} > F$ ) that will just break even. Overall, first investor loses
- Thus, it is crucial that investor extracts rent from financing second project. This is now feasible because successful entrepreneur has accumulated some wealth
- Bolton-Scharfstein: investor is monopolist and  $\overline{X} - \underline{X} < F$  (as in our case) so that self-financing not feasible

- Possible alternatives to monopolist story:
  - Cash-flows not contractible but projects are (veto power)
  - Variation: right to seize a crucial asset (maybe valueless)
  - If self-finance infeasible, sufficient that cash-flow transfers to new investors be contractible (seniority rule)
  - Investor has acquired unique information
- Rather than putting emphasis on threat to cut off funding, one may view investor as merely selling his veto-right to undertake second investment
- Given investor has all bargaining power, he can ask for price which leaves entrepreneur just indifferent:
  - price = entrepreneur's valuation

$$\bar{R} = p(\bar{X} - \underline{X}) + \underline{X}$$

- **Relationship to CSV:**

In both non-verifiable income models and in CSV models, investor has to undertake (sometimes) ex-post inefficient action in order to get repaid

- In non-verifiable income models, he denies financing for (second) positive NPV project
- In CSV models, he incurs wasteful verification costs

As a result of the wasteful activity, renegotiation is an issue in both type of models

- Note: Cost of inefficient action in instances where inefficient action is triggered, are borne (at least ex-post) by different parties
  - CSV: Verification cost borne by investor
  - Non-verifiable income: entrepreneur forgoes profitable second investment

Who bears the costs matters in a world where some agents are cash-constrained

- Bolton-Scharfstein type of model can easily be extended to multiple periods ( $\neq CSV$ ) (Gromb LSE94)
- Same intuition: "Development phase" followed by a "trading phase". Make a poor entrepreneur (country) rich and then sell him right to continue. The more valuable future credit is due to highly profitable projects, the higher is current repayment that investor can demand. Of course, investor may also have little incentive to shut down after a default, i.e., the termination threat may not be credible

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