APPENDIX

TO LECTURE NOTES ON

ASSET PRICING AND PORTFOLIO MANAGEMENT

2011

Professor B. Espen Eckbo

1. Portfolio analysis in Excel spreadsheet
2. Formula sheet
3. List of Additional Academic Articles
Notes on Portfolio Selection using Excel

These notes discuss how to find the optimal portfolio of risky assets using Excel (i.e. the point of tangency from the riskfree asset that gives an investor the best risk-return tradeoff).

The handout focuses on using vector notation to solve the problem. See BKM for a discussion of the problem.

1. Inputs

Main inputs: expected returns, variances and covariances. Standard (mathematical) way to summarize this information: vector of expected returns \( \mu \) and variance-covariance matrix \( \Sigma \).

The \((n \times 1)\) vector \( \mu \) has elements which are the expected returns of each of the assets. It is usually better to work with the vector of excess expected returns \( \mu^e \), where the riskfree rate is subtracted from the expected returns.

The variance-covariance matrix \( \Sigma \) is an \((n \times n)\) matrix that records the variances in the diagonal (i.e. the elements \( \Sigma_{ii} \)), and the covariances in the off-diagonal terms \( \Sigma_{ij} \). This is a symmetric matrix \( (\Sigma_{ij} = \Sigma_{ji}) \), and it is negative-semidefinite (which means that when you run a portfolio \( x \) through it as \( x^T \Sigma x \) you always get something that is non-negative (here superscript T denotes “transpose”).

**Example 1.** Expected return for three assets: 10%, 12%, 20%. Volatilities: 20% for all. Correlation between asset 1 and 2 is 0.50, and all other correlations are zero. Risk-free rate is 5%. Then

\[
0.05 \\
0.07 \\
0.15
\]

\[
\mu^e = 
\]

and

\[
0.04 \quad 0.02 \quad 0 \\
0.02 \quad 0.04 \quad 0 \\
0 \quad 0 \quad 0.04
\]

\[
\Sigma = 
\]

Note: 0.02 = 0.5(0.2)(0.2) is the covariance of assets 1 and 2.
II. Calculating means and variances

In order to calculate the expected excess return and variance of a portfolio $x$, 

$$E(r_x) - R_f = x^T \mu_e$$
$$\text{var}(r_x) = x^T \Sigma x$$

Excel is a bit tricky in order to do vector and matrix operations. Best thing is to keep vectors vertical and matrices square. Then another useful trick is to hit <Ctrl><Shift><Enter> once a formula has been entered.

In order to perform the calculations, suppose the portfolio $x$ is in cells H1:H3. If the variance-covariance matrix was in cells A1:C3, and the excess expected returns in E1:E3, the following two (intuitive) formulas would give as the expected excess return on $x$ and its variance:

$$\text{=mmult(transpose(H1:H3),E1:E3)}$$
$$\text{=mmult(mmult(transpose(H1:H3),A1:C3),H1:H3)}$$

Remember: you need to hit <Ctrl><Shift><Enter> once you entered the formula.

III. Optimal Portfolios

With no constraints on short-sales the solution to an investor’s problem involves investing in the portfolio

$$x = \Sigma^{-1} \mu_e \hspace{1cm} (1)$$

End of story.

Example 2. Equation (1) tells us that the optimal portfolio (an $n \times 1$ vector) is equal to the product of the inverse of $\Sigma$ and $\mu^e$.

In Excel, when performing an operation involving vectors and/or matrices you need to select the set of cells that are the solution before entering the formula (you did not need this before because the mean and variance of a portfolio were scalars, i.e. $1 \times 1$ vectors).

In our particular case this involves selecting a $3 \times 1$ area of cells. If the variance-covariance matrix was in cells A1:C3, and the excess expected returns in E1:E3, the command that gives you the answer is (don’t press Enter yet) is

$$\text{=mmult(minverse(A1:C3), E1:E3)}$$

after which we need to press: <Ctrl><Shift><Enter> (don’t ask me why).

It’s tricky following these steps the first time, but it is worth the time.
It’s also worth remembering that this is an intermediate step in the asset allocation process. Once we know what the optimal risky portfolio $x$ is, we still need to decide on the mix between the riskfree asset and this optimal risky portfolio.

IV Alternative Approach

Section 8.5 in BKM discusses another approach to constructing the mean-variance efficient frontier.

Note that in the previous discussion, we did not look for the whole frontier, but rather just for the optimal portfolio among those on the frontier (which will be combined with the riskfree asset).

It is not surprising that a similar spreadsheet as that in 8.5 should help us get the job done using Solver. As we should all know, the optimal risky portfolio solves

$$\max(x) \quad S_x = \frac{[E(r_x) - R_f]}{\sigma_x} = \frac{(x^T \mu^e)}{\sqrt{x^T \Sigma x}} \quad (2)$$

We know how to calculate the quantities in the numerator and denominator, so we can just use Solver.

It should be noted that using this “brute-force” approach by maximizing (2) is the only way to find a solution once we include constraints (such as short-sale constraints).
Formula Sheet

- Conversion between continuously compounded \((cc)\) and simple \((s)\) returns:
  \[
  r_s = \exp(r_{cc}) - 1 \quad \quad r_{cc} = \ln(1+r_s)
  \]

- Fraction of your wealth you put in the risk-free asset \(A\):
  
  \[
  w^* = \frac{E(r_A - r_F)}{A^2}
  \]

- The MVE portfolio weights when there are two risky assets \((x_2 = 1-x_1)\)
  
  \[
  x_A = \frac{[E(r^e_A)\sigma^2_B - E(r^e_B)\sigma_{AB}]}{[E(r^e_A)\sigma^2_B + E(r^e_B)\sigma^2_A] - [E(r^e_A) + E(r^e_B)\sigma_{AB}]}
  \]

- The CAPM equation
  
  \[
  E(r_i) = r_F + \beta_i[E(r_M) - r_F] \quad \text{where} \quad \beta_i = \frac{\sigma_{im}}{\sigma^2_M}
  \]

- The security characteristic line
  
  \[
  r_{it} - r_{Ft} = \alpha_i + \beta_i(r_{mt} - r_{Ft}) + \epsilon_{it}
  \]

- The systematic variance of a security
  
  CAPM: \(\sigma^2_{sys,i} = \beta^2_i\sigma^2_M\) (CAPM)
  
  APT: \(\sigma^2_{sys,i} = \sum_j \sum_k \beta_{ij} \beta_{ik} \sigma_{jk} = \sum_k \beta^2_{ik}\sigma^2_k\) when factors are uncorrelated

- The variance of portfolio \(a\) and covariance of portfolios \(a\) and \(b\)
  \[
  \sigma^2_a = \sum_i \sum_j x^a_i x^a_j \sigma_{ij} \quad \sigma_{ab} = \sum_i \sum_j x^a_i x^b_j \sigma_{ij}
  \]
  
  Two assets, 1 and 2:
  
  \[
  \sigma^2_a = (x^a_{1})^2 \sigma^2_1 + (x^a_{2})^2 \sigma^2_2 + 2x^a_{1}x^a_{2}\sigma_{12}
  \]

  \[
  \sigma_{ab} = x^a_{1}x^b_{1} \sigma^2_1 + x^a_{2}x^b_{2} \sigma^2_2 + (x^a_{1}x^b_{2} + x^a_{2}x^b_{1})\sigma_{12}
  \]

- The APT return generating process
\[ r_{it} = E(r_{it}) + \beta_{i1}f_{1t} + \beta_{i2}f_{2t} + \ldots + \beta_{ik}f_{kt} + \epsilon_{it} \]

- The APT pricing equation

\[ E(r) = \lambda_0 + \lambda_1\beta_{11} + \lambda_2\beta_{12} + \ldots + \lambda_k\beta_{k1} \]

- Managed fund performance measures

1. The Sharpe Ratio of portfolio \( p \)

\[ S_p = \frac{r^p}{\sigma_p} \]

2. Jensen’s Alpha is the \( \alpha_p \) from the regression

   \[ \text{(CAPM)} \quad r^p_t = \alpha_p + \beta_p r^f_{mt} + \epsilon_{pt} \]

   \[ \text{(APT)} \quad r^p_t = \alpha_p + \beta_{p1} r^f_{1t} + \beta_{p2} r^f_{2t} + \ldots + \beta_{pK} r^f_{Kt} + \epsilon_{pt} \]

   where \( r^f_{kt} \) is the excess return on the factor-mimicking portfolio for factor \( k \)

3. The Treynor Measure

\[ T_p = \frac{(E(r_p) - r_F)}{\beta_p} \]

or, adjusted (relative to the slope of the SML)

\[ T^*_p = \frac{(E(r_p) - r_F)}{\beta_p} - \frac{(E(r_M) - r_F)}{\beta_M} = \frac{\alpha_p}{\beta_p} \]

4. The Appraisal Ratio of portfolio \( p \) (CAPM and APT)

\[ AR_p = \frac{\alpha_p}{\sigma_{ep}} \]

Sharpe Ratio of optimal portfolio \( C \) of \( M \) and \( p \) is

\[ SR_C = (SR^2_M + AR^2_p)^{1/2} \]

5. The Henriksen-Merton Timing Measure is \( c_p \) from the regression

\[ r^p_t = \alpha_p + \beta_p r^f_{Mt} + D_t c_p r^x_{Mt} + \epsilon_{pt} \]

where \( D_t \) is a dummy variable that takes on a value of 1 when \( r_{Mt} > r_{Ft} \)

6. The Treynor-Mazuy Timing Measure is the coefficient \( c_p \) from the regression
(CAPM) \[ r_{pt} = \alpha_p + \beta_p r_{Mt} + c_p (r_{Mt})^2 + \epsilon_{pt} \]

(APT) \[ r_{pt} = \alpha_p + \beta_{p1} r_{1t} + \beta_{p2} r_{2t} + \ldots + \beta_{pK} r_{Kt} + c_{p1}(r_{1t})^2 + c_{p2}(r_{2t})^2 + \ldots + c_{pK}(r_{Kt})^2 + \epsilon_{pt} \]

The value of selectivity is \( \alpha_p \).

CAPM: The value of market timing is \( c_p \sigma_M^2 \)

APT: The total value of factor tilting is \( c_{p1} \sigma_1^2 + c_{p2} \sigma_2^2 + \ldots + c_{pK} \sigma_K^2 \)
List of Academic Articles

I. Stock Market Survival


II. Asset Pricing Basics


III. Return to Momentum Strategies


IV. Management Fees in Closed-End Funds


V. Mutual Fund Performance


VI. Performance and Flow of Funds


VII. Use of Derivatives by Mutual Funds


VIII. Closed-End Fund Discounts


IX. Hedge Funds


**X. Private Equity**


**XI. Pension Fund Activism**


**XII. Individual Investor Portfolios**


**XIII. Performance of New Issues**


23. **Eckbo and Norli**: “Leverage Liquidity and Long-Run IPO Returns”, working paper Tuck School at Dartmouth.

**XIV. Performance of Insider Trades**