Hiring and Firing Fund Managers

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This paper models the relationship between a fund manager's portfolio performance figures and an investor's decision, each period, to either retain or fire the fund manager. The investor must decide on the basis of evolving information whether to abandon the current manager, and at cost K, engage the best alternative manager. The model highlights the value of the real option to delay the decision by one or more periods, after which some uncertainty about the quality of the manager is resolved. In addition, the model partially explains why some investors wait a seemingly irrational length of time before changing manager and shows the effect of uncertainty over manager skill on the value of that real option. A closed form solution is presented for the case of an investor *n* periods after hiring the manager and 2 periods from the investment horizon. Simulation results are presented for the case of an investor with a 10 year investment horizon.

I. Introduction

Investors who opt for delegated portfolio management have a large number of professional fund managers to choose from, including active managers and passive managers. Investors use information available at the time that funds are invested to choose one or more fund managers. Over time new information arrives. At a cost, the investor can enact some combination of the following actions: dismissal of one or more of the fund managers and employment of new managers; re-balancing of the investment across the existing managers; and/or re-negotiation of the contractual terms with one or more managers.

In deciding which managers to choose, investors may have preferences over several characteristics of fund managers, including: size of funds under management; range of services provided; administrative support; and location of offices. However, a major consideration for all investors who choose active management is the ability of the fund manager to add value by attaining positive risk adjusted excess-to-benchmark returns.

Investors receive performance results at regular intervals. If the fund manager performs poorly, other things equal, the investor must decide whether to change for the next period the value of assets delegated to that manager.[1] In this study the following simple case is modelled. An investor chooses a single fund manager on the basis of the information available at time t_0. Performance results then arrive at the end of each period, at which time the investor decides whether to continue with the manager, or alternatively, incur cost K and choose a new manager. The investor's optimal manager dismissal policy is derived here for a two period model.

In this paper the investor's problem is placed in the specific context of a pension fund plan tionship with its single fund manager. There are advantages to choosing a pension fund context. Firstly, assumptions about risk, contractual form and costs can be made simple and reasonable. Secondly, the descriptive results of the modelling can be compared to known institutional arrangements in the pension fund industry.

Most of the performance measurement literature analyses problems associated with measures of asset selection or market timing abilities. Brealey and Hodges (1972) and Ashton (1996) take a different tack. They discuss the practical difficulty of getting enough observations to make inferences about measures of fund manager ability. Brealey and Hodges simulate optimal portfolio formation by managers who have various levels of stock selection and market timing

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[1] Contractual terms between investors and institutional or retail investment managers are rarely renegotiated (Lakonishok, Shleifer and Vishny 1992a, Golec 1992, Goode 1993).

ability. They show that for a manager who has a correlation between predicted stock return residuals and realised residuals of 0.15 (a high figure), approximately 100 quarters of data would be required to attain 95 percent confidence that the correlation is not zero. Ashton seeks an econometric lower bound for the number of observations required to discern plausible levels of manager forecasting ability. He similarly finds that over 100 periods of data are required.

But we know that plan sponsors do not wait 100 periods before making decisions. In fact the relationship between pension fund plan sponsors and fund managers is quite fluid. Brown, Davies and Draper (1992a) report that 56.1 percent of their plan sponsor survey respondents indicated that they changed their relationship with one or more of their fund managers in the previous 3 years.

The Brealey and Hodges, and Ashton approaches ignore two crucial aspects of plan sponsor behaviour. Firstly, levels of statistical certainty are incidental to the investor's decision. Rather, the investor has an objective, such as minimisation of the cost of a pension scheme, in which the choice of manager is a decision variable. The critical level of certainty about fund manager ability, at which the decision to change managers is made, is determined endogenously and varies with the circumstances of the problem - such as the cost of changing manager. Secondly, the investor has prior information about the manager's ability to add value. It is hard to imagine how an investor could choose a manager from a completely uninformed position.

The investor's decision process involves the combination of prior information with evolving sample information to choose optimally between mutually exclusive decisions (to retain or dismiss) which have stochastic payoff states. This is naturally a Bayesian updating decision problem. Naturally - because it seems indisputable that investors update their information through time and then choose optimally from a set of managers.

A two period model of the optimal manager dismissal policy is developed here. Heinkel and Stoughton (1994) develop a two period Bayesian updating model of manager performance. Their purpose, however, is to explain observed features of investor/fund manager contracts, such as flat rate fees, boiler plate contracts and independent performance measurement.

A model of an investor's rational fund manager dismissal policy has a number of potential applications. Firstly, in understanding incentives faced by fund managers. A better understanding of the structure of incentives faced by fund managers may help to explain features of the individual and collective behaviour of fund managers; such as, momentum investing, herding, index tracking and mean reversion of returns in 'universes' of fund managers. Most fund managers in the mutual fund and pension fund industries are employed on fixed fee contracts (Lakonishok, Shleifer and Vishny 1992a, Golec 1992, Goode 1993). Poor performance does not usually result in lower fees but it may lead to termination of the contract. The investor's dismissal policy relates fund manager performance to dismissal, which in turn characterises the downside risk faced by that investor's fund manager.

In addition, a manager dismissal model can illuminate dynamics in the overall relationship between pension fund trustees and pension fund managers. Brown, Harlow and Starks (1996) view the fund management industry as a tournament. The tournament 'winners' get the new funds flowing into the industry each year. The others gain no new investment money but also lose little. Sirri and Tufano (1998), Goetzmann, Greenwald and Huberman (1992), Capon, Fitzsimmons and Prince (1996) provide results which support this theory. Ippolito (1992) looks at how investors react to poor performance. Goetzmann and Peles (1997) seek to explain the seemingly unnatural length of time that investors retain poor performing managers, in a cognitive dissonance framework.[2] A manager dismissal policy provides an additional framework in which to examine these industry dynamics.

The remainder of the paper is arranged as follows. Section 2 presents assumptions of the model of hiring and firing and sets up the dynmamic programming problem, section 3 reduces the problem to a two period horizon to get a closed form solution for the value of active management to the investor, section 4 presents propostions from the two period model, section 5 presents the results of a simulation study of the hiring and firing decision where the investment horizon is 10 years, and section 6 concludes.

II. A model of investors' hiring and firing decisions

A. The structure and assumptions of the model

This model of optimal fund manager dismissal policy is developed in the context of a pension fund plan sponsor which delegates management of its assets to a single fund manager. At time t_0, which is the beginning of period 1, the investor chooses a manager from among a 'universe' of a large number of managers. At the end of each period the investor receives the portfolio performance figures for the period which include the benchmark return and the excess-to-benchmark return, as described below. The investor then decides whether to retain the manager

[2] See also Shefrin and Statman (1985) and Odean (1998a) for discussion of the disposition of private investors to hold on to poor performing investments for too long.

for at least one more period, or alternatively, dismiss the manager and choose the best alternative manager from the universe of managers. The assumptions of the model are as follows.

A.1 *The plan sponsor's sole objective in choosing a fund manager is the minimisation of the cost of meeting pension fund liabilities.*

A.2 *The return on the pension fund's portfolio in period n has two components; a benchmark return and an excess-to-benchmark return. The excess-to-benchmark returns are assumed to be normally distributed about mean* m *with precision* t*, and the values of those parameters vary across fund managers.*

Equation 1

$$r_{pn} = r_{bn} + x_n$$

$$x_n \sim N(\mu,\tau)\ ,\quad n \in \{1, 2, \ldots\}$$

Where r_{pn} = return on the portfolio in period n (the period beginning at time t_{n-1})

r_{bn} = return on the benchmark portfolio in period n

x_n = the excess-to-benchmark return in period n

μ = the mean of the excess-to-benchmark return distribution of the incumbent manager

τ = the precision of the excess-to-benchmark return distribution of the incumbent manager.

All returns are after fees, expenses and turnover costs.

A.3 *The portfolio benchmark is chosen such that the excess-to-benchmark returns are independent of the factors that determine asset returns.*

A.4 *The stochastic process generating each manager's excess-to-benchmark returns is assumed to have time invariant parameters.*

By assumption A.1, the plan sponsor seeks to minimise the present value of the cashflows from the firm to the pension fund. There is a one-to-one correspondence between increases in the excess-to-benchmark return and reduction in the cashflows to the pension fund.

The excess-to-benchmark return is a metric for value added by the manager, and therefore should not reflect exposure to priced factors. The benchmark portfolio return can be formed as the fitted or explained return in a factor model that includes all priced factors. The excess-to-benchmark return is then simply the residual of the factor model estimation. Therefore, the average of excess-to-benchmark returns can be interpreted as a multi-factor version of Jensen's alpha (Jensen 1968). In which case, the excess-to-benchmark return is seeking to measure value added by stock selection rather than factor or market timing by the manager.[3] The importance of the assumption that the excess-to-benchmark return is independent of priced risk factors is that it allows the considerable simplification of discounting of expected excess-to-benchmark returns at the risk free rate.

The probability distribution of the benchmark return is chosen by the plan sponsor. The distribution of the excess-to-benchmark return is specific to fund managers. Each manager is fully characterised by a stochastic process for the manager's excess-to-benchmark portfolio conditioned by the benchmark set by the plan sponsor. Each realisation of that process is a draw from a normal distribution which has time invariant parameters.

Let $\theta^{\mathbf{i}} = (\mu^i,\tau^i)$ be the mean and precision of the distribution of excess-to-benchmark returns for manager *i*. There is a large group of fund managers from which the plan sponsor may choose. From the plan sponsor's perspective the managers vary only in the mean and precision of their excess-to-benchmark returns over the benchmark portfolio; reflecting the managers' different investment abilities, incentives and risk preferences. The plan sponsor aims to appoint the manager that is expected, over time, to add the most value to the funds under management.

Implicit to assumption A.4, of time invariance of the manager's excess-to-benchmark return distribution, is an assumption that managers are not strategically varying the distribution parameters μ and τ through time as their realised performance changes their incentives. This assumption also means that the dilution of fund manager's 'private information per dollar invested', resulting from new funds flowing to well performed fund managers, has no effect upon the mean of the excess-to-benchmark return distribution.

The plan sponsor uses all the initially available information to form a prior probability distribution over $\theta^{\mathbf{i}} = (\mu^i, \tau^i)$ for each manager *i* at time t_0. The available information includes the investment performance history of the manager, but might also include other information, such as

[3] Until recently there was little evidence of market timing ability among institutional investors. Studies by Ferson and Schadt (1996) and Chance and Hemler (1998) find evidence of timing ability.

recent personnel changes or investment in new technologies. In this Bayesian view, $\theta^{\mathbf{i}}$ is a random vector.

From the plan sponsor's perspective the universe of managers represents a set of projects, where -to-benchmark return in a period is the cash flow from the project which is that manager. At the end of any period the plan sponsor can switch from one project (manager) to another at a known cost of K per dollar invested. From the prior distribution on $\theta^{\mathbf{i}}$, denoted $\varsigma(\theta)$, the plan sponsor is able to value the anticipated stream of excess-to-benchmark returns, and manager change costs, which follow the choice of manager *i* to manage the pension fund portfolio from time t_0. This value, denoted V_0^i is the value of active management to the plan sponsor at time t_0 conditional upon the initial choice of manager *i*. It is determined in equation 2 under the following assumptions.

A.5 *The market value of the pension fund portfolio is re-balanced to $1 at the beginning of each period.*

A.6 *The income and assets of the pension fund are not taxed.*

A.7 *The interest rate on risk free assets is constant through time.*

Equation 2

$$V_0^i = \sum_{n=1}^{\infty} \frac{E\left[x_n^j \middle| \Im_0, I_0 = i\right] - K \, \mathrm{Prob}\left[D_n = 1 \middle| \Im_0, I_0 = i\right]}{(1+r)^n}$$

Where

V_0^i = the value of portfolio management to the plan sponsor at time t_0, conditional on the choice of manager *i* at t_0

E = expectation operator applied at time t_0

x_n^j = excess-to-benchmark return of the incumbent manager j in the period beginning at time t_{n-1} (until manager *i* is dismissed *j*=*i*).

$\Im_0$ = the plan sponsor's information set at time t_0

I_0 = the identity of the chosen manager at time t_0

K = the cost of changing manager per £ in portfolio value

D_n = the indicator function which takes value 1 if the plan sponsor changes manager at time t time t_n

r = the risk free interest rate.

Equation 2 characterises the valuation of each manager as a net present value problem. V_0^i is the value to the plan sponsor of active management conditioned by the plan sponsor's information set at time t_0 and the choice of manager *i* at time t_0. It is the discounted sum of the value added to the portfolio by active management in each period. That value added is the excess-to-benchmark return of the incumbent manager less any cost incurred in changing manager at the end of the period. The equation recognises that the identity of the incumbent manager may change.

The plan sponsor can associate a value V_0^i with each manager *i* in the universe of managers under consideration. After ranking the available managers, the plan sponsor chooses the manager which maximises the value of active management of the plan sponsor's portfolio.[4]

A.8 The expected value of the best alternative manager at the end of any future period is only infinitesimally below that of the highest valued manager at time t_0.

Equation 3

$$V_0 = \underset{\{i \in I\}}{\text{Max}}\ V_0^i = E\left[V_{n+m}^a \middle| \Im_n\right] \qquad n, m \in \{1, 2, \ldots\}$$

Where V_0 = the value of portfolio management to the plan sponsor at time t_0

V_0^i = the value of portfolio management to the plan sponsor at time t_0, conditional on the choice of manager *i*

V^a_{n+m} = the value of active management to the plan sponsor, at time t_{n+m}, if the best alternative manager is appointed at that time

$\Im_n$ = the plan sponsor's information set at time t_n

I = the set (universe) of all managers under consideration.

4 Passive management can be approximately characterised by an excess-to-benchmark return which has a known mean of zero and variance of zero. The manager's problem is trivial in this case - the value of index management is simply zero. If the

Assumption A.8 ensures that the plan sponsor makes decisions on the basis of the mathematical expectation that, in any period, if the current manager is dismissed, then after reassessing the universe of other managers the value of the chosen manager will be V^a.

The chosen manager is offered and accepts a flat fee contract. The terms of the contract are not renegotiated at any point. Having chosen a manager, the plan sponsor receives performance results at the end of each period.

If after n periods there has been no change of manager then the plan sponsor has observed a vector of excess-to-benchmark returns $\mathbf{x} = (x_1,....,x_n)$. Let V_n^r denote the value of active management at time t_n conditional upon retention of the manager for at least one more period. The value of active management at time t_n if the manager is dismissed is V^a-K. The value of active management at time t_n is then the maximum of V_n^r and V^a-K.

A.9 The only changes in the plan sponsor's information set are the performance figures of the chosen manager.

Equation 4

$$V_n = \text{Max}\,[V_n^r, V^a - K\,]$$

Where V_n = the value of portfolio management to the plan sponsor at time t_n

V_n^r = the value of portfolio management to the plan sponsor conditioned upon the existing manager being retained for at least one more period

V^a = the value of portfolio management conditioned on the dismissal of the existing manager and appointment of the best alternative manager.

B. Illustration of the value of active management

B.1 Manager with permanent tenure

Much of this paper is concerned with modelling the determinants of the value of active management to the investor. To begin, consider a plan sponsor that is choosing a manager who

trustee board does not assign a positive value to the management of any of the active portfolio managers then passive management is chosen and there is no reason to change manager in any future period.

after appointment will then have permanent tenure. In this case the choice of manager is a simple net present value problem.

Equation 5

$$V_0 = E\left[\frac{x_1}{1+r} \,\middle|\, \Im_0\right] + E\left[\frac{x_2}{(1+r)^2} \,\middle|\, \Im_0\right] + \ldots..$$

After n periods, during which $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ is observed, the posterior probability distribution over $\theta = (\mu,\tau)$, denoted $\xi_n(\theta)$, can be determined. Let m_n denote the plan sponsor's expected value of μ at time t_n.[5]

Equation 6

$$E\left[x_{n+1} \middle| \Im_n\right] = \int_{-\infty}^{\infty}\int_{0}^{\infty} E\left[x_{n+1} \middle| \mu,\tau\right]\xi_n(\mu,\tau)\,d\tau\, d\mu$$

$$= \int_{-\infty}^{\infty}\int_{0}^{\infty} \mu\, \xi_n(\mu,\tau)\,d\tau\, d\mu$$

$$= E\left[\mu \middle| \Im_n\right]$$

$$= m_n$$

The expected value of the excess-to-benchmark return, in the period beginning at t_n, is the expected value of μ over its posterior distribution at time t_n.

[5] The subscript *i* has been dropped to simplify notation.

Equation 7

$$
\begin{aligned}
E\left[x_{n+2} \mid \Im_n\right] &= E\left[E\left[x_{n+2} \mid \Im_{n+1}\right] \mid \Im_n\right] \\
&= E\left[m_{n+1} \mid \Im_n\right] \\
&= E\left[\int_\theta \mu\,\xi_{n+1}(\theta)\,d\theta \mid \Im_n\right] \\
&= \int_{-\infty}^{\infty}\int_\theta \mu\,\xi_{n+1}(\theta)\,d\theta\, f(x_{n+1})\,dx_{n+1} \\
&= \int_{-\infty}^{\infty}\int_\theta \mu \frac{\xi_n(\theta)\, f(x_{n+1} \mid \theta)}{f(x_{n+1})}\,d\theta\, f(x_{n+1})\,dx_{n+1} \\
&= \int_\theta \mu\,\xi_n(\theta) \int_{-\infty}^{\infty} f(x_{n+1} \mid \theta)\,dx_{n+1}\,d\theta \\
&= E\left[\mu \mid \Im_n\right] \\
&= m_n
\end{aligned}
$$

Therefore from equations 6 and 7 and mathematical induction we have:

Equation 8

$$E\left[x_1 \mid \Im_0\right] = E\left[x_2 \mid \Im_0\right] = \ldots .= m_0$$

$$V_0 = \sum_{n=1}^{\infty} E\left[\frac{x_n}{(1+r)^n} \mid \Im_0\right]$$

$$= \frac{m_0}{r}$$

where $\{x_1, \Im_0\}, \{x_2, \Im_1\}, \ldots$ forms a martingale

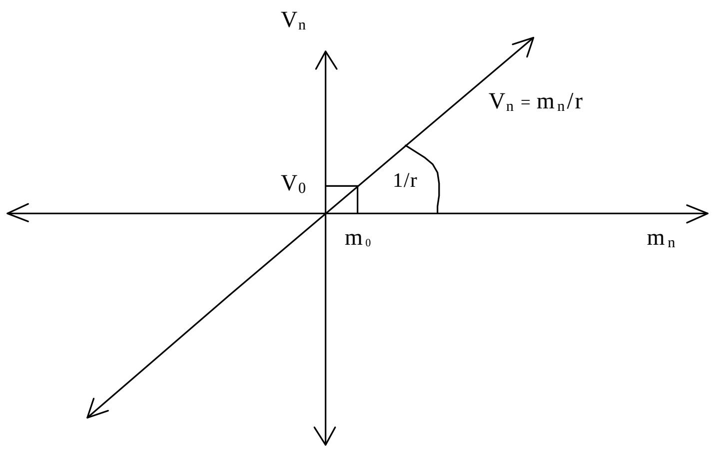
similarly, $V_n = \frac{m_n}{r}$

Under the assumption of permanent tenure for the chosen fund manager the value of portfolio management can be depicted in a two dimensional diagram. Figure 1 shows that the initial value is V_0 and a plan sponsor will only choose active management over passive management if $m_0 > 0$. That is, active management is chosen over passive management if the expected value of the mean of the excess-to-benchmark return distribution of the chosen manager is greater than zero. After the manager is chosen the vector of excess-to-benchmark returns evolves period by period. V_0 and V_n are related by the equation 9.

Equation 9

$$\text{If } \sum_{k=1}^{n} \frac{x_k}{n} \begin{matrix} < \\ = \\ > \end{matrix} m_0 \quad \text{then} \quad V_n \begin{matrix} < \\ = \\ > \end{matrix} V_0$$

Figure 1 Manager with permanent tenure



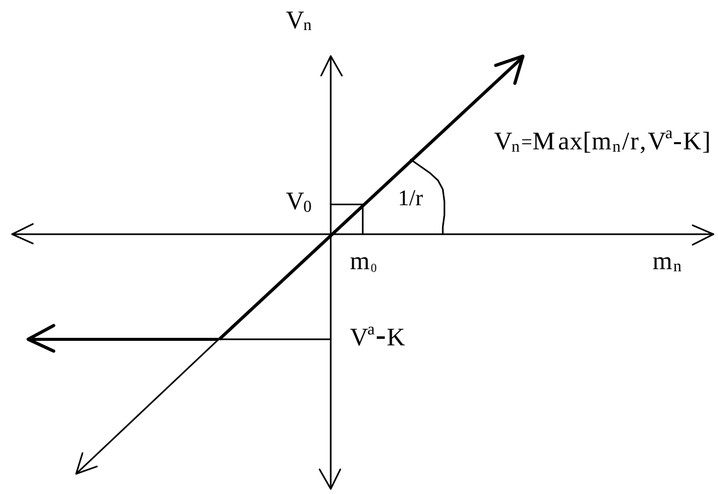
B.2 Option to change manager

Now assume that the plan sponsor has a one-off option to dismiss the manager at time t_n and choose a new manager, and then the new manager or retained manager has permanent tenure. The value of delegated portfolio management at time t_n is then the maximum of either the value conditioned on retaining the existing manager, denoted V_n^r, or the value conditioned on dismissing the existing manager which is the value of the best alternative manager, denoted V^a, less the cost K of switching to the best alternative manager.

Equation 10

$$V_n = \text{Max}\left[V_n^r \,,\, V^a - K\right] = \text{Max}\left(\frac{m_n}{r} \,,\, V^a - K\right)$$

Figure 2 Option to change manager in current period only



B.3 Option to change the manager at any time

The option at t_n, to dismiss the incumbent manager and employ the best alternative manager, puts a floor under the value of active management at that time. Now consider the conditions that plan sponsors actually face – the option to change manager at the end of any period. Now V_n^r denotes the value of delegated portfolio management conditional on retaining the manager for *one or more* periods.

Where the plan sponsor can dismiss the manager at any point in time, the decision at the end of each period is between two actions. Firstly, to choose a new manager at cost K. Or alternatively, to postpone the decision to change manager to the end of the next period at which time another observation from the fund manager's excess-to-benchmark return is available. This decision is analogous to that of an investor deciding whether to pay a known cost to abandon one project and commence another project. The 'real' option, to delay the abandonment decision, by one or more periods, has value. The value derives from the resolution, during the period of delay, of some uncertainty over a variable that affects the value of the project.

As with options on financial assets, insight can be obtained by considering limiting cases. Here the limiting case is that of the sample mean of the excess-to-benchmark return approaching positive infinity or approaching –1. In either case the value of the option to decide in future periods approaches zero.

Equation 11

$$V_n^r = \frac{E\left[x_{n+1} + V_{n+1} \middle| \Im_n\right]}{1+r}$$

$$= \frac{E\left[x_{n+1} \middle| \Im_n\right]}{1+r} + \frac{E\left[Max\left[V_{n+1}^r, V^a - K\right] \middle| \Im_n\right]}{1+r}$$

$$= \frac{m_n}{1+r} + \frac{E\left[Max\left[V_{n+1}^r, V^a - K\right] \middle| \Im_n\right]}{1+r}$$

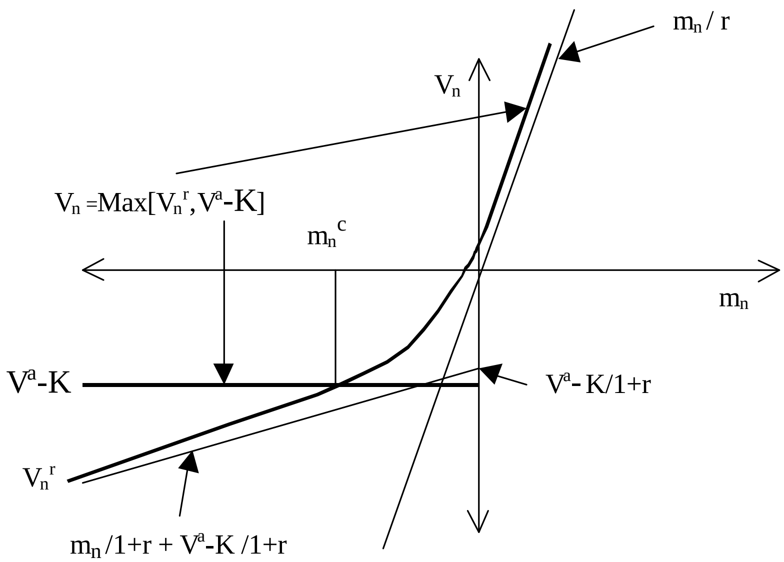
Equation 11 shows that, conditional on the retention of the existing manager for at least one more period, the value of delegated portfolio management is the sum of two quantities. The first is the discounted value of the expected excess-to-benchmark return in the next period. The second is the expected value of delegated portfolio management at the end of the next period discounted to the present. For V_n^r as a function of m_n, the asymptotes as m_n goes to $+\infty$ or -1 are easily deduced.

Equation 12

$$\lim_{m_n \to \infty} V_n^r = \frac{m_n}{r}$$

$$\lim_{\substack{m_n \to -1 \\ n \to \infty}} V_n^r = \frac{m_n}{1+r} + \frac{V^a - K}{1+r}$$

Figure 3 Option to change the manager at any time



As m_n goes to $+\infty$ investor sample information dominates any prior information. The investor's subjective probability of observing a sequence of excess-to-benchmark returns that would lead to dismissal of the existing manager goes to zero. This implies that the value of the option to dismiss the manager in any future period goes to zero, and the manager effectively has permanent tenure. Therefore, $V_n^r \rightarrow m_n/r$ from above (because the option goes to zero from above).

As m_n goes to -1 and n goes to ∞ the probability that the excess-to-benchmark return in the next period will be sufficient to prevent dismissal of the manager at time t_{n+1} approaches zero. Therefore the value of the option, to retain the manager for one more period after the next period, goes to zero. For finite n, the probability that the manager is dismissed next period is finite. For finite n we have a lower bound rather than a lower asymptote for V_n^r.

Figure 3 depicts the upper asymptote and lower bound of V_n^r. A curve is fitted to the asymptotes for illustrative purposes.[6] The vertical distance between the asymptotes and the V_n^r curve represents the option value of delegated portfolio management. The value of m_n below which the

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[6] Note that figure 6.3 is drawn with all variables that affect V_n, other than m_n, held constant. In particular parameters of the posterior distribution of θ that characterize the uncertainty over μ are held constant.

manager is dismissed at time t_n is m_n^c. As in the classic investment under uncertainty problem, the required rate of return on the project before the plan sponsor acts is higher when value of the option to delay the decision is included. The result is $m_n^c < r(V^a - K)$.

B.4 The Bellman functional equation

Continuing the expansion of the expression for the value of delegated portfolio management we have the following equation.

Equation 13

$$V_n = \mathrm{Max}\left[V_n^r, V^a - K\right]$$

$$V_n^r = \frac{E\left[x_{n+1} + V_{n+1} \mid \Im_n\right]}{1+r}$$

$$= \frac{m_n}{1+r} + \frac{E\left[\mathrm{Max}\left[V_{n+1}^r, V^a - K\right] \mid \Im_n\right]}{1+r}$$

$$= \frac{m_n}{1+r} + \frac{E\left\{\mathrm{Max}\left\{\frac{E\left[x_{n+2} + V_{n+2} \mid \Im_{n+1}\right]}{1+r}, V^a - K\right\} \mid \Im_n\right\}}{1+r}$$

This expression is a recursive Bellman equation which can be expanded indefinitely. The plan sponsor faces a dynamic programming problem. They can choose to retain the manager or acquire more information. The option to delay now is an option on all the future options to delay and hence the expression has an infinite expansion.

V_n^r and V_{n+1}^r are functions which are related by the functional equation 13. The plan sponsor wishes to compare V^a - K, which is known, to V_n^r which is a function of **x** the vector of observed excess-to-benchmark returns. From equation 11 we can see that to solve for V_n^r we need to know the functional form of V_{n+1}^r. We can proceed to a solution by guessing V_{n+1}^r and then solving for V_n^r, then using this form as the next guess for V_{n+1}^r. Convergence is guaranteed, under modest regularity conditions, by the Contraction Mapping Theorem of functional equations.[7] An equivalent solution can be obtained by assuming a termination time when the fund is wound up and working backwards recursively.

III. A two period model

To solve the functional equation which determines the value of delegated portfolio management, a functional form of the prior distribution over θ must be specified. It is assumed here that the parameters of the manager's excess-to-benchmark return, $\theta = (\mu,\tau)$ have a gamma-normal distribution. The normal distribution of the excess-to-benchmark return, $x \sim N[\mu,\tau]$, and the gamma-normal distribution of the parameters of that normal distribution, $\theta = (\mu,\tau)$, form a Bayesian conjugate pair. This means that when the sample data drawn from the normal distribution is combined with the gamma-normal prior, the resulting posterior distribution is also gamma-normal.

In the course of this analysis several Bayesian conjugate pairs were considered for the modelling of the return distribution and the prior, including, the binomial--beta conjugate pair; the exponential--gamma conjugate pair; the Poisson--gamma conjugate pair; and the normal--gamma-normal conjugate pair. None of the available conjugate pairs leads to a closed form solution for V_n where the plan sponsor's investment horizon is infinite. If assumptions are made about the parameters of prior distribution, the interest rate r and the cost of changing managers K, then a numerical solution for V_n can be obtained. The next section presents the simulation results.

In this section, to pursue an understanding of the relationship between variables in the plan sponsor's problem, a closed form solution is obtained by reducing the plan sponsor's investment horizon to two periods from the decision date t_n. In the two period model, the plan sponsor is making the dismissal decision n periods after the incumbent manager was initially appointed. Therefore $\mathbf{x}_n=(x_1,x_2,..,x_n)$ has been observed at the decision point. Further, the investment horizon time is t_{n+2}. Meaning that the plan sponsor must decide at time t_n whether to retain or dismiss the manager, then make the equivalent decision at time t_{n+1}, and then the fund is wound up at time t_{n+2}. Figure 4 depicts the decision timing.

Figure 4 **Decision times**

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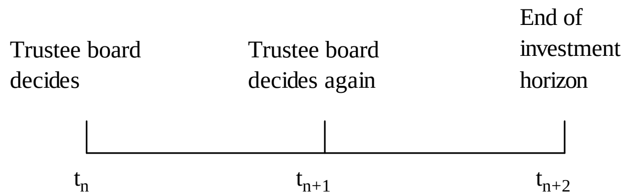
7 See Dixit and Pindyck, (1994, p102), or Stokey and Lucas (1993, p50).



To proceed we need to assume a form for the prior distribution over the mean and precision of the excess-to-benchmark return distribution. Assume that θ has a gamma-normal distribution. Also known as the gamma inverse normal distribution, where variance takes the place of precision. See DeGroot (1989, p402) for the standard gamma-normal distribution results given here.

Where $\xi o(\theta)$ in the prior distribution are over θ.

Equation 14

$$\xi_0(\theta) = \xi_0(\mu, \tau) = \xi_0(\tau)\xi_0(\mu | \tau)$$

The parameters of the gamma-normal prior distribution are α_0, β_0, λ_0 and m_0.

Equation 15

$$\xi_0(\theta) = \frac{\beta_0^{\alpha_0} \tau^{\alpha_0 - 1} e^{-\beta_0 \tau}}{\Gamma(\alpha_0)} \sqrt{\frac{\lambda_0 \tau}{2\pi}} \mathrm{e}^{\frac{-\lambda_0 \tau}{2}(\mu - m_0)^2}$$

The marginal distribution of τ is a gamma function with parameters α_0 and β_0.

Equation 16

$$\xi(\tau) = \frac{\beta_0^{\alpha_0}\ \tau^{\alpha_0 - 1}\ \mathrm{e}^{-\beta_0 \tau}}{\Gamma(\alpha_0)}$$

The conditional distribution of μ is a normal distribution with mean m_0 and precision $\lambda_0\tau$.

Equation 17

$$\xi(\mu | \tau) = \sqrt{\frac{\lambda_0 \tau}{2\pi}}\ \mathrm{e}^{\frac{-\lambda_0 \tau}{2}(\mu - m_0)^2}$$

The marginal distribution of μ can be obtained from a student t distribution with $2\alpha_0$ degrees of freedom. It is the student t distribution shifted to have a

Equation 18

mean of m_0 and scaled by $\left(\frac{b_0}{l_0 a_0}\right)^{\frac{1}{2}}$

$$\frac{m - m_0}{\left(\frac{b_0}{l_0 a_0}\right)^{\frac{1}{2}}} \sim t_{2a_0}$$

Where dof $= 2\alpha_0$

$E[\mu \mid \Im_0] = m_0$

Var $[\mu \mid \Im_0] = \frac{b_0}{l_0 a_0}$

$\beta_o > 0, \alpha_o \geq 1, \lambda_o > 0$

The gamma-normal Bayesian conjugate pair is suitable for this analysis for several reasons. Firstly, in the gamma-normal distribution the information is combined in an intuitively appealing way, as shown below in the explanation of how the parameters update. Another appealing property of the gamma normal distribution is the relationship between the precision of the excess-to-benchmark return x_n and the precision with which μ is known. For a given τ the precision of μ is $\lambda_o\tau$. This means that if $\lambda_0>1$, then the less noise there is in observations of the manager's excess-to-benchmark returns (higher τ), the more precise will be the prior over μ (higher $\lambda_0\tau$). Finally, the gamma normal distribution has a flexible shape which can accommodate a wide range of plausible prior beliefs.

To establish the prior distribution, the plan sponsor chooses α_0, β_0, λ_0 and m_0 such that:

The mean of the marginal μ distribution $= m_0$

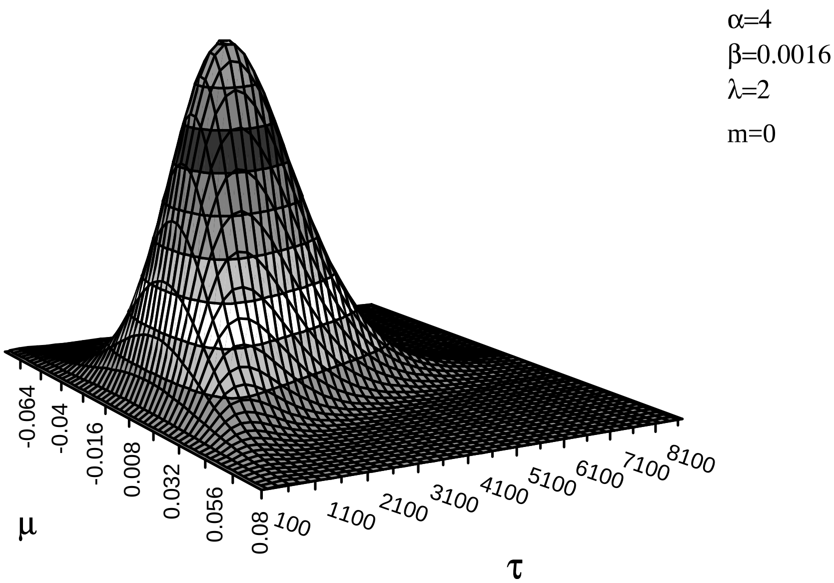
The variance of the marginal μ distribution $= \frac{b_0}{l_0 a_0}$

The mean of the marginal τ distribution $= \frac{\alpha_0}{\beta_0}$

The variance of the marginal τ distribution $= \frac{\alpha_0}{\beta_0^2}$

The gamma normal prior is graphed in figure 5. The parameter values of the depicted distribution are α=4, β=0.0016, λ=2 and m=0. These are the parameters used in the base case for the simulation study of the next section.

Figure 5 The gamma normal prior distribution



A. The gamma-normal posterior distribution

As noted previously the posterior is also a gamma normal distribution.

Equation 19

$$\xi_n(\theta \mid \mathbf{x}) = \frac{\beta_n^{\alpha_n} \tau^{\alpha_n - 1} e^{-\beta_n \tau}}{\Gamma(\alpha_n)} \sqrt{\frac{\lambda_n \tau}{2\pi}}\, e^{\frac{-\lambda_n \tau}{2}(\mu - m_n)^2}$$

Where $\overline{x}_n = \frac{\sum_{i=1}^{n} x_i}{n}$

$$s_n^2 = \frac{\sum_{i=1}^{n} (x_i - \overline{x}_n)^2}{n-1}$$

$$m_n = \frac{n\overline{x}_n + \lambda_0 m_0}{\lambda_0 + n}$$

$$\lambda_n = \lambda_0 + n$$

$$\alpha_n = \alpha_0 + \frac{n}{2}$$

$$\beta_n = \beta_0 + \frac{n-1}{2} s_n^2 + \frac{n\lambda_0(\overline{x}_n - m_0)^2}{2(\lambda_0 + n)}$$

The process of updating parameters is highly intuitive. The expected value of μ after excess-to-benchmark returns have been observed, denoted m_n, is a weighted average of the sample and prior information. Each weight reflects the amount of information contributed by that source. The parameter λ_0 controls the balance between the prior and sample information in determining m_n.

The Bayesian predictive distribution of the unknown x_{n+1} can be obtained as follows.

Equation 20

$$f(x_{n+1}\,|\,\mathbf{x}) = \int_{\theta} f(x_{n+1}\,|\,\theta, \mathbf{x})\, \xi_n(\theta\,|\,\mathbf{x})\, d\theta$$

$$= \int_{-\infty}^{\infty}\int_{0}^{\infty} \sqrt{\frac{\tau}{2\pi}}\, e^{-\frac{\tau}{2}(x_{n+1}-\mu)^2}\, \frac{\beta_n^{\alpha_n}\, \tau^{\alpha_n - 1}\, e^{-\beta_n \tau}}{\Gamma(\alpha_n)}\, \sqrt{\frac{\lambda_n \tau}{2\pi}}\, e^{\frac{-\lambda_n \tau}{2}(\mu - m_n)^2}\, d\tau\, d\mu$$

Gathering the τ terms and integrating yields the following expression.

$$f(x_{n+1}\,|\,\mathbf{x}) = \int_{-\infty}^{\infty} \frac{\sqrt{\lambda_n}\, \beta_n^{\alpha_n}\, \Gamma(\alpha_n + 1)}{2\pi\, \Gamma(\alpha_n)}\, \frac{2^{\alpha_n + 1}}{\left[2\beta_n + (x_{n+1} - \mu)^2 + \lambda_n(\mu - m_n)^2\right]^{\alpha_n + 1}}\, d\mu$$

A useful substitution is:

$$(x_{n+1} - \mu)^2 + \lambda_n(\mu - m_n)^2 = \frac{\lambda_n (x_{n+1} - m_n)^2}{\lambda_{n+1}} + \lambda_{n+1}(\mu - m_{n+1})^2$$

$$\text{Where } m_{n+1} = \frac{x_{n+1} + \lambda_n \mu_n}{\lambda_{n+1}}$$

By making the substitution and then grouping the τ terms into a student t form and integrating them out, the following expression is found.

Equation 21

$$f(x_{n+1} \mid \mathbf{x}) = \frac{\Gamma\left(\alpha_n + \frac{1}{2}\right)}{\Gamma(\alpha_n)} \frac{\left(1 + \dfrac{(x_{n+1} - m_n)^2}{2\alpha_n \left(\dfrac{\lambda_{n+1}\beta_n}{\lambda_n \alpha_n}\right)}\right)^{-\left(\alpha_n + \frac{1}{2}\right)}}{\sqrt{2\alpha_n \pi}\sqrt{\dfrac{\lambda_{n+1}\beta_n}{\lambda_n \alpha_n}}}$$

Equation 21 shows that x_{n+1} has the distribution of a student t with $2\alpha_n$ degrees of freedom, which has been shifted by m_n and scaled by $\left(\frac{\lambda_{n+1}\beta_n}{\lambda_n \alpha_n}\right)^{\frac{1}{2}}$. The predicted excess-to-benchmark return in the next period has a variance which is λ_{n+1} times that of the variance of μ.

$$\frac{x_{n+1} - m_n}{\sqrt{\dfrac{\lambda_{n+1}\beta_n}{\lambda_n \alpha_n}}} \sim t_{2\alpha_n}$$

Where: $\text{dof} = 2\alpha_n$; $E[x_{n+1} \mid \Im_n] = m_n$, $\text{Var}[x_{n+1} \mid \Im_n] = \dfrac{\lambda_{n+1}\beta_n}{\lambda_n \alpha_n}$

B. Decomposition of the value of delegated portfolio management

Returning to the plan sponsor's decision process, if $V_n^r > V^a - K$ then the manager is retained. Otherwise the alternative manager is contracted at cost K.

Equation 22

$$V_n^r = E\left[\frac{x_{n+1} + V_{n+1}}{1+r} \,\middle|\, \Im_n\right]$$

$$= E\left[\frac{x_{n+1}}{1+r} + \frac{\max\left[V_{n+1}^r\,,\ V_{n+1}^a - K\right]}{1+r} \,\middle|\, \Im_n\right]$$

$$= \frac{m_n}{1+r} + \frac{\int_{-\infty}^{\infty} \max\left[V_{n+1}^r\,,\ V_{n+1}^a - K\right] f(x_{n+1})\, dx_{n+1}}{1+r}$$

$$= \frac{m_n}{1+r} + \frac{V_{n+1}^a - K}{1+r} + \frac{\int_{x_{n+1}^c}^{\infty} \left[V_{n+1}^r - (V_{n+1}^a - K)\right] f(x_{n+1})\, dx_{n+1}}{1+r}$$

V_n^r is a function of x_{n+1}. The integration limit x^c_{n+1} is the critical value of x_{n+1} below which the manager is dismissed at time t_{n+1}, if the manager is retained at time t_n. If $x_{n+1} < x^c_{n+1}$ then the manager will be dismissed at the end of period n+1. Otherwise the manager is retained at that time. In this two period model x^c_{n+1} is a function of m_n. A value for x^c_{n+1} is found as follows.

At x^{c}_{n+1}

$$V^{r}_{n+1} = V^{a}_{n+1} - K$$

$$\frac{E\left[x_{n+2} \mid \Im_{n+1}\right]}{1+r} = \frac{m^{a}_{0}}{1+r} - K$$

$$\frac{m_{n+1}}{1+r} = \frac{m^{a}_{0} - K(1+r)}{1+r}$$

$$\frac{x^{c}_{n+1} + l_{n} m_{n}}{l_{n+1}} = m^{a}_{0} - K(1+r)$$

$$x^{c}_{n+1} = m_{n} + l_{n+1}\left\{m^{a}_{0} - K(1+r) - m_{n}\right\}$$

Where m^{a}_{0} = the mean of μ^{a} in the prior distribution of the best alternative manager.

Equation 23 can be rearranged to give the components of the value of delegated portfolio management.

Equation 23

$$V^{r}_{n} = \frac{m_{n}}{1+r} + \frac{V^{a}_{n+1} - K}{1+r} + \frac{\int_{x^{c}_{n+1}}^{\infty}\left[V^{r}_{n+1} - \left(V^{a}_{n+1} - K\right)\right] f(x_{n+1})\, dx_{n+1}}{1+r}$$

$$= \frac{m_{n}}{1+r} + \frac{V^{a}_{n+1} - K}{1+r} + \frac{\int_{x^{c}_{n+1}}^{\infty}\left\{\frac{m_{n+1}}{1+r} - \left(V^{a}_{n+1} - K\right)\right\} f(x_{n+1})\, dx_{n+1}}{1+r}$$

$$= \frac{m_{n}}{1+r} + \frac{V^{a}_{n+1} - K}{1+r} + \int_{x^{c}_{n+1}}^{\infty}\left\{\frac{x_{n+1} + l_{n} m_{n}}{l_{n+1}(1+r)^{2}} - \frac{V^{a}_{n+1} - K}{1+r}\right\} f(x_{n+1})\, dx_{n+1}$$

$$= \frac{m_n}{1+r} + \frac{V^a_{n+1} - K}{1+r} F\left(x^c_{n+1}\right) - \frac{m_n}{\left(1+r\right)^2}\left[1 - F\left(x^c_{n+1}\right)\right] + \int_{x^c_{n+1}}^{\infty} \left(\frac{x_{n+1} - m_n}{l_{n+1}\left(1+r\right)^2}\right) f\left(x_{n+1}\right) dx_{n+1}$$

$$= \frac{m_n}{1+r} + \frac{m^a_0 - K\left(1+r\right)}{\left(1+r\right)^2} F\left(x^c_{n+1}\right) + \frac{m_n}{\left(1+r\right)^2}\left[1 - F\left(x^c_{n+1}\right)\right]$$

$$+ \frac{1}{l_{n+1}\left(1+r\right)^2} \int_{x^c_{n+1}}^{\infty} \left(x_{n+1} - m_n\right) \frac{\Gamma\left(a_n + 0.5\right)}{\Gamma\left(a_n\right)} \frac{1}{\sqrt{2pa_n}\sqrt{\frac{l_{n+1}b_n}{l_n a_n}}} \left(1 + \frac{\left(x_{n+1} - m_n\right)^2}{2a_n\left(\frac{l_{n+1}b_n}{l_n a_n}\right)}\right)^{-\left(a_n + 0.5\right)} dx_{n+1}$$

The integral is the value of an European option with and a strike price of x^c_{n+1}-m_n. In this option the probability distribution of the underlying asset is a student t shifted to have a mean of m_n and scaled to have a variance $\left(\frac{l_{n+1}b_n}{l_n a_n}\right)$. Integrating and rearranging yields the following decomposition of the value of delegated portfolio management conditional on the incumbent being retained for at least one more period.

Equation 24

$$V^r_n\left(\overline{x}_n, s^2_n, n\right) = \frac{m_n}{1+r} + \frac{\left(m^a_0 - K\left(1+r\right)\right)}{\left(1+r\right)^2} F\left(x^c_{n+1}\right) + \frac{m_n}{\left(1+r\right)^2}\left[1 - F\left(x^c_{n+1}\right)\right]$$

$$+ \frac{1}{\left(1+r\right)^2 l_{n+1}} \left(\frac{l_{n+1} b_n}{l_n\left(a_n - 1\right)}\right) t_{2a_n - 2}\left[x^c_{n+1}\right]$$

Where F( ) = the distribution function of xn+1

$\overline{x}_n$ = the sample mean of observed excess-to-benchmark returns

s_n^2 = sample variance of excess-to-benchmark returns

n = the number of periods since appointment of incumbent manager

To determine whether to retain the manager the plan sponsor calculates V_n^r and compares this value to V^a-K. Where V^a is calculated using the μ_0^a, λ_0^a, β_o^a and α_0^a parameters of the best alternative manager, with $\overline{x}_n = s_n^2 = n = 0$. Equation 24 can be interpreted as follows. At time t_n the value of active management conditional upon retention of the manager for at least another period has four components:

i. The discounted expected excess return next period.

$$\frac{m_n}{1+r}$$

ii. The discounted expected excess return in the second period conditional upon changing manager at the end of next period, times the probability of that event.

$$\frac{m_0^a - K(1+r)}{(1+r)^2} F(x_{n+1}^c)$$

iii. The discounted expected excess return in the second period conditional upon retaining the manager at the end of the next period, times the probability of that event.

$$\frac{m_n}{(1+r)^2}\left[1 - F(x_{n+1}^c)\right]$$

iv. The value of the option to decide whether to retain or change after the next period's excess return information is received.

$$\frac{1}{\lambda_{n+1}(1+r)^2}\left(\frac{\lambda_{n+1}\beta_n}{\lambda_n(\alpha_n-1)}\right) t_{2\alpha_n-2}\left[x^c_{n+1}\right]$$

V_n^r is a function of : $\bar{x}_n$ the sample mean of the observed excess-to-benchmark returns; s_n^2 the sample variance; n the number of observations. Only the first two sample moments are important - as might be expected from a prior distribution based upon normal and gamma functions.

IV. Implications of a dismissal policy

The plan sponsor's valuation of portfolio management after n periods is a function of parameters, constants and state variables.

α_0, β_0, λ_0, m_0	The parameters of the prior distribution over θ.
k, r	The constants which are exogenous to the plan sponsor's decision making.
$\bar{x}_n$, s_n^2, n	The state variables of the dynamic programming problem faced by the plan sponsor.

The comparative statics of these variables can provide insight into various influences upon fund manager dismissal decisions. The two period gamma-normal model of fund manager dismissal supports the following propositions.

A. Proposition 1:

If the vector of reported excess-to-benchmark returns is $\mathbf{x}_n = (x_1, ..., x_n)$*, then* $\bar{x}_n$, s_n^2 *and n are sufficient to determine* V_n .

This follows directly from equation 24 and the value rule V_n = Max [V_n^r , V^a-K] . The result implies that, conditional upon a gamma-normal prior, plan sponsors need only know the number of performance observations and the first two moments of those observations to attain the value of active management after n periods. It also implies that the value process is Markovian rather than path dependent, because of the Bayesian nature of the decision process.

B. Proposition 2:

The value of active management is non decreasing in uncertainty over the expected value of the manager's excess return.

$\frac{dV_n}{d\sigma_m^2} \geq 0$ *where* $\sigma_m^2 = \frac{\beta_n}{\lambda_n \alpha_n}$ is the variance of the expected excess-to-benchmark return after observing $\mathbf{x_n} = (x_1, ..., x_n)$

If the investor retains the manager for period n+1, then there is a unique value of x_{n+1}, denoted x^c_{n+1}, for which the investor is indifferent between retaining or dismissing the manager at the end of that period. Equation 24 can be rearranged to express V_n^r in an option form.

Equation 25

If $x^c_{n+1} > m_n$ then

$$V_n^r = \frac{m_n}{1+r} + \frac{V^a - K}{(1+r)} + \frac{1}{\lambda_{n+1}(1+r)^2}\int_{x^c_{n+1}}^{\infty}\left(x_{n+1} - x^c_{n+1}\right) f(x_{n+1})\, dx_{n+1}$$

If $x^c_{n+1} < m_n$ then

$$V_n^r = \frac{m_n}{1+r} + \frac{m_n}{(1+r)^2} + \frac{1}{\lambda_{n+1}(1+r)^2}\int_{-\infty}^{x^c_{n+1}}\left(x^c_{n+1} - x_{n+1}\right) f(x_{n+1})\, dx_{n+1}$$

In the first expression the integral term represents the value of a European call option with a strike price at x^c_{n+1}, over an asset, the value of which has a t distribution with mean m_n and variance $\lambda_{n+1}\sigma_u^2$ at the strike date, which is one period hence. In the second expression the integral term represents the value of the corresponding put option. Since it is well known that these option terms are strictly increasing in the variance of the value of the underlying asset and since none of the other terms are functions of that variance we know that:

V_n^r is increasing in $l_{n+1} s_m^2$, the variance of x_{n+1}

$$\frac{d\,V_n}{d\,s_m^2} \geq 0 \quad \text{since} \quad V_n = \text{Max}\left[V_n^r, V^a - K\right]$$

The uncertainty over the expectation of the excess-to-benchmark returns is measured by the variance of m which is a function of β_n, α_n and λ_n. These parameters therefore combine to represent the uncertainty over the value that the manager can add to the delegated portfolio. The value of active management is increasing in this uncertainty.

This proposition partially explains two questions arising from empirical research into the flow of investors' funds to and from investment management groups. Firstly, it is found by Ippolito (1992), Brown, Harlow, Starks (1996), Goetzmann and Peles (1997), Sirri and Tufano (1998) and others, that fund managers can return persistently poor performance relative to their peers without experiencing large outflows of money. These results have been explained in terms of high switching costs, including the tax costs of realising capital gains. Nonetheless, relative to perceived costs of changing manager, the poor performance that some investors tolerate has no obvious full explanation in the literature.

The framework for analysing investors hiring and firing decisions presented here recognises that a delay in the decision to switch managers may allow resolution of some uncertainty over the manager's quality. As in a real options framework, the more uncertainty that can be resolved by delay the higher is the required return to the new project (the worse the manager's performance) before the new project is initiated.

Secondly, this proposition partially explains an observation of Chevalier and Ellison (1997a). They found that after controlling for size and for the performance of managers, the growth rate of the funds managed by managers with shorter performance histories is greater than equivalent managers with longer histories. A shorter history equates to less prior information, which in this model is captured by a lower value for λ_0. The greater uncertainty of the expected value of the excess-to-benchmark returns of the managers with shorter performance records makes them more valuable to investors who discount excess-to-benchmark returns at the risk free rate.

A final point relates to the entry of fund managers to the industry. The number of fund managers entering (and exiting) the fund management industry is large - particularly in the mutual fund industry. This observation may be partially explained by the uncertainty about μ, the expected excess return for managers new to the industry. Because investors are more uncertain about the new fund managers, and since investors have the option to sack the manager at low cost and choose another, relatively unknown managers may have a comparative advantage.

C. Proposition 3:

The value of active management is non decreasing in the shape parameter β_0 of the gamma-normal prior distribution or the sample variance of the excess return observations.

$$\frac{d\,V_n}{d\,\beta_0} \geq 0 \ , \quad \frac{d\,V_n}{d\,s_n^2} \geq 0$$

This is true because $\frac{d\,V_n}{\sigma_m^2} \geq 0$, β_0 and s_n^2 enter V_n only through the expression

$$\sigma_u^2 = \frac{\beta_0 + \left(\frac{n-1}{2}\right)s_n^2 + \frac{\lambda_0 n_0 (\bar{x} - m_0)^2}{\lambda_0 + n}}{(\lambda_0 + n)\left(\alpha_0 + \frac{n}{2}\right)}$$

The value of active management is non decreasing in the sample variance. This result has significance for active managers who deliberately track the market index. Index tracking can assure that the sample mean of excess-to-benchmark returns is close to zero but it also reduces the sample variance which in turn reduces V_n^r. A manager that replicates the benchmark portfolio may be more likely to exceed x^c_{n+1}; however, index tracking decreases the variance of their excess-to-benchmark returns, which decreases V_n^r and in turn increases x^c_{n+1} .

In this framework managers have an incentive to inject noise into their return to increase the plan sponsor's valuation. This idea is closely related to the Dow and Gorton (1997) noise and volume argument. They argue that managers with no information will undertake noise trading so that their lack of information is not revealed in the short term.

D. Proposition 4:

The value of active management is decreasing in K, the cost of changing managers, and not increasing in r the risk free interest rate.

$$\frac{\partial V_n}{\partial k} < 0 \quad , \quad \frac{\partial V_n}{\partial r} \leq 0$$

From equation 23 the value of active management conditional on retaining the manager can be expressed as:

$$V_n^r = \frac{m_n}{1+r} + \frac{m_n}{(1+r)^2} + \frac{1}{l_{n+1}(1+r)^2}\int_{-\infty}^{x_{n+1}^c} \left[x_{n+1}^c - x_{n+1}\right] f(x_{n+1})\, dx_{n+1}$$

$$\text{Since } \frac{d\left[V_{n+1}^a - K\right]}{dK} < 0 \ , \quad \frac{d\, x_{n+1}^c}{dK} = -(l_{n+1})(1+r) < 0$$

$$\frac{dV_n^r}{dK} < 0 \quad \text{and therefore } \frac{dV_n}{dK} < 0$$

$$\text{Similarly, } \frac{d\, x_{n+1}^c}{dr} = -K(l_{n+1}) < 0$$

$$\frac{dV_n^r}{dr} < 0 \quad \text{and therefore } \frac{dV_n}{dr} \leq 0$$

A consequence of this result is that active investment managers may create externalities for their industry if they increase costs of changing manager. The active investment management industry competes with passive investment management for funds. If investors can only estimate an industry value of K, rather than a manager specific value, then the externality may lead to higher costs of changing manager and thereby the value of active management to investors is decreased relative to passive management.

For example, when a portfolio is passed from a dismissed manager to a newly appointed manager, the costs of trading to the new manager's preferred portfolio holdings is typically not attributed to either the old or new manager. For large pension funds this unattributed cost may be the largest component of the cost of changing manager. Because of the lack of attribution managers have little incentive to minimise this cost, but the value of active management to an investor is strictly decreasing in the investor's estimate of this cost.

The relationship between the value of active management and the riskless rate simply presents a testable implication of this model. In a large universe of managers, the number of investors switching managers, over any short period, should be increasing in r.

E. Proposition 5:

There is not a simple monotonic relationship between the value of active management and either the volume of prior information, measured by λ_0, or the volume of sample information, measured by n.

$$\text{If}\quad \bar{x} > \mu_0 \ \text{and}\ s_n^2 > \frac{\beta_0}{\alpha_0 \lambda_0} \ \text{then}\ \frac{\partial V_n^r}{\partial n} > 0$$

$$\text{If}\quad \bar{x} < \mu_0 \ \text{and}\ s_n^2 < \frac{\beta_0}{\alpha_0 \lambda_0} \ \text{then}\ \frac{\partial V_n^r}{\partial n} < 0$$

$$\text{Otherwise}\quad \frac{\partial V_n^r}{\partial n} \ \frac{\leq}{\geq}\ 0$$

V_n is a function of μ_n and σ_n^2. μ_n is a weighted average of $\bar{x}_n$ the sample mean and μ_0 the prior mean. The weights are $\frac{n}{\lambda_n}$ and $\frac{\lambda_0}{\lambda_n}$ respectively. If $\bar{x}_n > \mu_0$ then an increase in n puts greater weight on the sample data. The same reasoning applies to s_n^2 and $\frac{\beta_0}{\alpha_0\lambda_0}$. If the sample mean and variance are consistent with high value then shifting weight to sample information increases V_n^r. Both the sample mean and variance are important.

This result characterises the simple idea that a larger number of observations of performance is good news for managers whose performance has exceeded the investor's prior expectation of the manager's average excess-to-benchmark return. However, uncertainty over the mean of the manager's excess-to-benchmark distribution is also an important determinant of the manager's value to the investor. If more sample information increases the posterior expected value of μ, but decreases the uncertainty over μ then the effect of n on V_n is ambiguous. Note that the same reasoning applies to λ_0 because λ_0 and n enter the expression for V_n^r symmetrically.

F. Stationarity and equilibrium

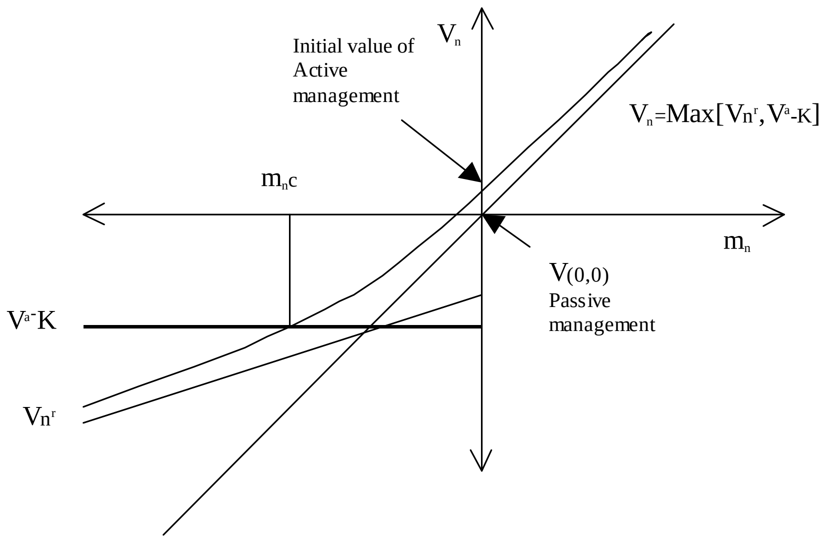
It is assumed here that the processes that generate the fund manager's excess-to-benchmark returns are stationary. However, the assumption implies that the fund manager does not react strategically as the vector of performance results evolves. The fund manager will learn about the plan sponsor's prior over the fund manager's excess return through: the initial fund manager selection process; the benchmarks that are set; and the ongoing contact with the plan sponsor. Presumably the fund manager will use an informed assumption about the plan sponsor's prior to formulate a strategic response.

A manager's estimate of x^c_{n+1} represents the size of the 'bets' that the manager can afford to take. If a manager has a given level of information on the future realisation of an asset price, then the quantity of that asset held will be a function of x^c_{n+1}. A manager can take bets on specific assets and at the same time insure against a portfolio return of less than $x^c_{n+1.}$ Many pension fund management mandates preclude the purchase of derivative securities. Instead, the manager can hedge by dynamically adjusting the holding of the benchmark portfolio and individual assets.

The parameter stability assumption implies the existence of performance persistence. There is not much evidence of performance persistence in the pension fund industry (Lakonishok, Shleifer and Vishny 1992a, Brown, Draper and M^cKenzie 1997 and Brown, Goetzmann, Ibbotson and Ross 1992). Performance persistence studies face severe methodological problems in handling survivorship bias and risk adjustment.

If the excess return distributions of managers are stationary, then we might expect to see a migration of pension funds from poor managers to good managers over time. In the UK pension fund industry over the last 15 years, pension fund investment mandates have become concentrated in the hands of the most successful investment management firms. This observation is consistent with a degree of stationarity.

Figure 6 Active versus passive management



G. Passive and active management

In the framework of this paper a passive manager can be characterised by

$$\theta^{index} = (\mu , \tau) = (0,0). \quad \Rightarrow \quad V_0^{index} = 0$$

Passive management provides a calibration of the value of active management. This sets a lower bound on V^a-K of $-K^{index}$. Each plan sponsor's assessment of V^a-K determines their choice between active and passive management.

The plan sponsor depicted in figure 6 initially chooses active management even though $m_0 < 0$. This illustrates the point that a plan sponsor may choose active management even if they do not expect on average to choose a manager with positive μ. Under the conditions of this model plan sponsors which choose active management do not rationally have to believe that they will on average choose a manager who will beat the market - they just have to believe that they can hold on to managers who do beat the market for longer. If the plan sponsor has a rational view of the market then they must believe that they have higher than average ability to choose a manager with a high μ. This is because over time active fund managers on average have lower returns than passive managers (Lakonishok, Shleifer and Vishny 1992b, Brown, Draper and McKenzie 1997).

Little is known about the factors that determine the fraction of the total capitalisation of a financial asset market that is under passive management. The figure must be less than 100 percent to permit the rational formation of prices (Grossman and Stiglitz 1980). However, the literature says little else. The framework presented here partially characterises the equilibrium between passive and active management (institutional or private). Plan sponsors which believe that V^a-K > -K^{index} will see another active manager as the best alternative to their current manager; otherwise they will go to passive management when they change manager. As the volume of passively managed funds rises, V^a increases (in a Grossman Stiglitz world) - giving rise to the equilibrium. Since 1986 the pension fund industry in the UK has seen both a slow drift of pension fund mandates to passive management and a narrowing of the average returns of the active management and passive management sectors.

V. A simulation study

A.1 Purpose of the simulation

The two period model developed in section 3 yields a closed form solution for the value of active management which in turn yields the propositions of section 4. However, investors typically have horizons longer than two years. In the case of plan sponsors the investment horizon is often the duration of the liabilities: which for a young workforce may be several decades.

The model cannot be extended by increasing the horizon without giving up either the closed form nature of solutions or the gamma – normal Bayesian structure of the model. A simpler Bayesian model may have a closed form, but such a model may be too abstract to be of interest. The alternative to a closed form solution is a numerical solution. In this section the results of numerical solution of the value of active management for a plan sponsor with an investment horizon of 10 years are presented.[8]

There are three objectives in this study. Firstly, to estimate the size of the option effects for investors with longer horizons. The two period model indicates that option effects exist but does not demonstrate that they have economic significance. Secondly, to confirm the direction of the comparative static results set out in the propositions of section 4. Thirdly, to determine the economic significance of each of these comparative static results.

The numerical solution of the model of hiring and firing for differing parameter values amounts to a simulation study of the model. That is not a substitute for confronting the testable implications of the model with data. However, simulation is useful and necessary to guide the formation of interesting testable implications before proceeding with the task of collecting data for empirical testing.

A.2 Nature of simulation

The simulation proceeds by numerically solving the following equation.

Equation 26

$$V(\bar{x}, s^2, n \mid \alpha_0, \beta_0, \lambda_0, m_0, r, K)$$

$$= \frac{m_n}{1+r} + \int_{-\infty}^{+\infty} \frac{\text{Max}\left[V(\bar{x}(x_{n+1}), s^2(x_{n+1}), n+1 \mid \alpha_0, \beta_0, \lambda_0, m_0, r, K), V(0, 0, 0) - K\right]}{1+r}$$

This is a discrete functional equation in V(.). The chosen values of α_0, β_0, λ_0, m_0, k, r specify the prior distribution and the exogenous economic variables. The value of the best alternative manager is calculated as V(0, 0 ,0), the initial value of the existing manager. Recall that it is assumed here that the value of the best alternative manager is invariant with time. A solution is a value of V(.) at each point in the state space of points ($\bar{x}$, s_n^2, n). In this simulation the approximation to the full solution is the values of V(.) at the points:

---

8 My discussions with pension fund consultants indicated that 10 years is often chosen as the time to the investment horizon in their simulation studies.

n = 1, 2, 3, ……., 20

$\bar{x}$ = -0.100, -0.099, ……, 0,…….0.098, 0.099

s_n^2 = 0.0000, 0.0002,…...,0.0098

Therefore, the value of active management at 20 x 200 x 50 = 200,000 points are estimated in each simulation. Those values are the source of data for the graphs and tables presented in this section. The simulations compute the value of $V(\bar{x}, s_n^2, n)$ for an investor who has an investment horizon of 10 years. For some situations a longer horizon might be appropriate. However, the number of computations in the simulation increases as the square of the number of periods to the horizon. In any case we know from the Bayesian structure of the model that the results will be qualitatively similar for larger horizons.[9]

B. Simulation methodology

The simulation can proceed to a solution by solving for $V(\bar{x}, s_n^2, n)$ where the horizon is one period. In that case $V(\bar{x}, s_n^2, n) = \text{Max}\,[m_n/(1+r),(m_0^a-K(1+r))/(1+r)]$ where m_0^a is the expected return from the best alternative manager over the one period. Having solved for the one period values, the simulation can step back a period and use the 200,000 values from the one period problem to calculate the values for the two period horizon. Then the simulation iterates back another eight times to reach the ten year horizon solution.

In each iteration for each of the 200,000 cells, the integral in equation 27 must be evaluated. In this study, Montecarlo estimation of the integral was employed. For each cell calculation a large number of independent draws are made from the student t distribution of x_{n+1} that relates to that cell. The value in the next period that corresponds to the draw is found and the values are averaged. This method appeals to the law of large numbers to estimate the expectation of V(.) over the possible values of the excess-to-benchmark return in the next period.

9 The number of computations required in the simulation if the reporting periods are quarters instead of years is 16 times greater.

B.1 Calibration of the Montecarlo simulation

Two elements of the Montecarlo simulation are central to its accuracy. Firstly, the range of the points calculated and the distance between those points.[10] Secondly, the number of draws in evaluation of the integral. To determine suitable values for these quantities the Montecarlo simulation values can be calibrated against the closed form solution from the two period model.

For the two period horizon case, the value of $V(\bar{x}, s_n^2, n)$ was calculated by both the closed form equation of 24 and the Montecarlo method. The results of this calibration are depicted in figures 7 and 8. Figure 7 shows that the closed form and Montecarlo values are co-incident. In the legend to that figure hor=2 means that these results are for the two period horizon model. Draw = 5000 records that 5000 draws were made in the Montecarlo estimation.

Figure 8 depicts the value of the option component of V_n^r (part 4 in equation 24), as it varies with the sample mean. The different lines show the effect of increasing the number of draws in the Montecarlo estimation technique. The results suggest that the number of cells (200,000) is large enough and 1,000 draws is sufficient for the Montecarlo method.[11]

C. Representative investor case

C.1 Parameter choices

The values of α_0, β_0, λ_0 and m_0 determine the slope and location of the prior distribution. In this simulation the parameters were chosen on the following basis.

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[10] Outside the range the value is estimated by the upper asymptote for high sample mean values which are outside the range of cells and V^a-K for the low sample mean values which are outside the range of cells.

[11] This calibration also permits very accurate de-bugging of the simulation program which provides a non trivial improvement in confidence in the simulation results.

$\frac{a_0}{b_0} = E(\tau)$ Values of $\alpha_0 = 4$ and $\beta_0=0.0016$ were chosen to give an expected value of τ of 2500 in the prior.

$\alpha_0 = \text{dof}$ A prior degree of freedom in the t distribution of x_{n+1} of 4 was chosen. The choice of α_0 also determines the variance of τ which is α_0/β_0^2.

λ_0 Determines the weight that is given to prior information versus sample information in forming the posterior distribution. The choice of $\lambda_0 = 2$ corresponds to a representative investor whose prior information is equivalent to two observations (years) of sample information.

m_0 In this simulation the representative investor is given a prior expected value of μ of 0.

Figures 9 – 12 depict the value of active management to the representative plan sponsor when the investment horizon is 10 years and the investment manager has been employed for 3 years. Figure 9 shows the value of active management for the representative plan sponsor for different values of the sample variance of excess-to-benchmark returns. It shows that $V(\bar{x}, s_n^2, n)$ is increasing in the sample variance. In particular the figure demonstrates that the value of $\bar{x}$ at which the manager is dismissed is decreasing in the sample variance.

Figure 9 shows the low and high asymptotes which V_n^r approaches when the sample mean of excess-to-benchmark returns is low or high respectively. Figure 10 shows the difference between V_n^r and the asymptotes. This difference represents a part of the value of the option to delay the decision to dismiss the manager by one or more periods. Figure 10 shows that part of the option value is an increasing function of s_n^2. Much of the option value is captured by V^a because the option to change managers stills exists after a new manager is chosen.

Figure 11 exhibits the relationship between V_n^r and the number of observations of the excess-to-benchmark return. Clearly there is not a single monotonic relationship between the volume of sample information and the value of active management. The position of the asymptotes is dependent upon n. Nonetheless, figure 12 shows that the height of V_n^r above the asymptotes (the option value not captured in V^a) is a decreasing function of n. That option value is increasing in the level of uncertainty over μ, which is decreasing in n.

D. Uncertainty over the expected value of excess-to-benchmark returns

Tables 1 and 2 and figures 13 and 14 depict the effect of the level of uncertainty over μ on the value of active management. The variance of μ is $\sigma_\mu^2 = \frac{\beta_n}{\lambda_n \alpha_n}$. In the simulations for this sub-section the effect of varying σ_μ^2 was studied by varying β_0 and keeping the initial degrees of freedom (α_0) and the weight on prior information (λ_0) constant.

The middle row of each of the following cases presents data for the representative investor, as discussed in the previous section. Table 1 shows that for the representative investor the minimum value of $\bar{x}$ at which the manager is dismissed at the decision date is decreasing in s_n^2. The value falls 50 basis points between $s_n^2 = 0.0002$ and $s_n^2 = 0.0050$.

For higher and lower values of uncertainty over μ, this range is much the same. However, table 1 shows that the critical value of $\bar{x}$ at which the investor is indifferent between retention or dismissal is decreasing in β_0. Investors with high values of β_0, that is, high uncertainty over the value of μ, will accept a lower sample mean of excess-to-benchmark returns before dismissing the manager. Another important effect of varying σ_μ^2 is to shift the value of the alternative manager and hence the value of active management. This is particularly apparent in table 2 which shows the value of active management for different values of β_0 and s_n^2 when $\bar{x} = 0$.

Table 1 Effect of uncertainty over μ on dismissal level of $\bar{x}$

	Sample variance			
	s_n^2=0.0002	s_n^2=0.0010	s_n^2=0.0024	s_n^2=0.0050
$\beta_0 = 0.0064$	0.005	0.005	0.002	0.000
$\beta_0 = 0.0016$	-0.003	-0.004	-0.005	-0.008
$\beta_0 = 0.0004$	-0.006	-0.008	-0.010	-0.012

Table 2 Effect of uncertainty over μ on value of active management at $\bar{x}$=0

	Sample variance			
	s_n^2=0.0002	s_n^2=0.0010	s_n^2=0.0024	s_n^2=0.0050
β_0 = 0.0064	0.0426	0.0425	0.0456	0.0483
β_0 = 0.0016	0.0101	0.0125	0.0154	0.0204
β_0 = 0.0004	-0.0134	0.0022	0.0036	0.0132

E. Level of prior information

Simulation results are presented in tables 3 and 4 and figures 15 and 16 where weight on the prior information versus sample information is varied, with other parameters held constant. Higher values of λ_0 are consonant with giving more weight to the prior information in the formation of the posterior distribution. The critical value of $\bar{x}$ for which the investor is indifferent between retention and dismissal is decreasing in the weight placed on prior information. Moreover at $\bar{x}$=0 the value of active management is strongly decreasing in the weight put on prior information.

Table 3 Effect of level of prior information on dismissal level of $\bar{x}$

	Sample variance			
	s_n^2=0.0002	s_n^2=0.0010	s_n^2=0.0024	s_n^2=0.0050
λ_0 = 4	-0.008	-0.009	-0.011	-0.014
λ_0 = 2	-0.003	-0.004	-0.005	-0.008
λ_0 = 1	0.004	0.002	0.000	-0.002

Table 0.4 Effect of level of prior information on value of active management at $\bar{x}$=0

	Sample variance			
	s_n^2=0.0002	s_n^2=0.0010	s_n^2=0.0024	s_n^2=0.0050
λ_0 = 4	0.0040	0.0043	0.0075	0.0106
λ_0 = 2	0.0101	0.0125	0.0154	0.0204
λ_0 = 1	0.0271	0.0283	0.0327	0.0377

F. Effect of the cost of changing manager

Tables 5 and 6 and figures 17 and 18 depict the effect of the cost of changing fund manager on the value of active management. It is clear from table 6 that the value of active management is falling in the cost of changing manager. However, the reduction in value is less than the increase in cost. Comparing the columns of table 6, the results suggest that the ratio of decrease in value of active management to increase in cost of changing is increasing with the sample variance. Thus, the more likely it is that a change is required, the larger the effect of K on V(.).

Table 5 suggests, as expected, that investors will accept a lower value of $\bar{x}$ before changing manager when the cost of changing manager is higher.

Table 5 Effect of K on dismissal level of $\bar{x}$

	Sample variance			
	s_n^2=0.0002	s_n^2=0.0010	s_n^2=0.0024	s_n^2=0.0050
K=0.04	-0.010	-0.012	-0.013	-0.015
K=0.02	-0.003	-0.004	-0.005	-0.008
K=0.01	0.001	0.000	-0.001	-0.004

Table 6 Effect of K on value of active management at $\bar{x}$=0

	Sample variance			
	s_n^2=0.0002	s_n^2=0.0010	s_n^2=0.0024	s_n^2=0.0050
K=0.04	0.0029	0.0031	0.0067	0.0099
K=0.02	0.0101	0.0125	0.0154	0.0204
K=0.01	0.0217	0.0228	0.0266	0.0308

G. Effect of the risk free rate

Tables 7 and 8 and figures 19 and 20 depict the effect of the cost of changing fund manager on the value of active management. Table 7 indicates that investors will accept lower values of $\bar{x}$ before dismissing a manager when interest rates are high. The future payoffs to switching managers is lower when interests rates are higher. However, the value of active management is strongly decreasing with increased interest rates at a sample mean of 0. In this framework active management is less valuable when interest rates are high but nonetheless investors have less incentive to change managers.

Table 7 Effect of risk free rate on dismissal level of $\bar{x}$

	Sample variance			
	s_n^2=0.0002	s_n^2=0.0010	s_n^2=0.0024	s_n^2=0.0050
r=0.20	-0.008	-0.009	-0.010	-0.012
r=0.08	-0.003	-0.004	-0.005	-0.008
r=0.02	-0.001	-0.001	-0.003	-0.007

Table 8 Effect of risk free rate on value of active management at $\bar{x}$=0

	Sample variance			
	s_n^2=0.0002	s_n^2=0.0010	s_n^2=0.0024	s_n^2=0.0050
r=0.20	0.0029	0.0032	0.0056	0.0078
r=0.08	0.0101	0.0125	0.0154	0.0204
r=0.02	0.0210	0.0227	0.0288	0.0350

VI. Concluding remarks

The model of hiring and firing of pension fund managers and the associated simulation study are motivated by a desire to understand how the decisions of investors, regarding which fund manager to choose and whether to retain that manager, affect the equilibrium in asset markets. Investors' decisions on dismissal can help us to understand the downside risk that fund managers face from poor portfolio performance results. An understanding of fund manager incentives is crucial in studying the market equilibrium.

Empirical studies find that significant poor under-performance by fund managers may be sustained before the volume of funds under management decreases substantially. In the model presented here, that observation is explained by the positive value of the option to delay the decision to dismiss the manager.

In the two period model, the value of active management can be broken down into four components, one of which is the option to decide next period, rather than this period, whether to dismiss the manager. The simulation of the value of active management to an investor with an investment horizon of 10 periods shows that this option value is not trivial. The simulation draws out the crucial role of the level of uncertainty over the expected value of the manager's excess return distribution. In the simulation the difference between a high prior level of uncertainty and a low prior level is nearly 500 basis points in the value of the manager, and 110 basis points in the sample mean of excess-to-benchmark returns before the manager is dismissed. For a longer horizon these values would be commensurately larger.

This level of uncertainty over μ is also increasing in the sample variance of excess-to-benchmark returns, which has implications for the incentives of fund managers who are approaching performance levels at which investors are considering dismissal.

The model highlights the role of the cost of changing manager in the decisions of investors. The simulation shows the value of active management decreasing in the cost of changing managers with a sensitivity of less than one. The model also potentially provides insights into the role of the cost of changing manager in the balance between the total volume of funds under active versus passive management. Individual managers who have performed poorly do not have the same incentives as highly performed managers to minimise the costs to their investors of changing manager. Since the value of active management, unlike passive management, is decreasing in the cost of changing manager, there is a negative externality associated with these costs.

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Figure 7 Closed form values versus Montecarlo values

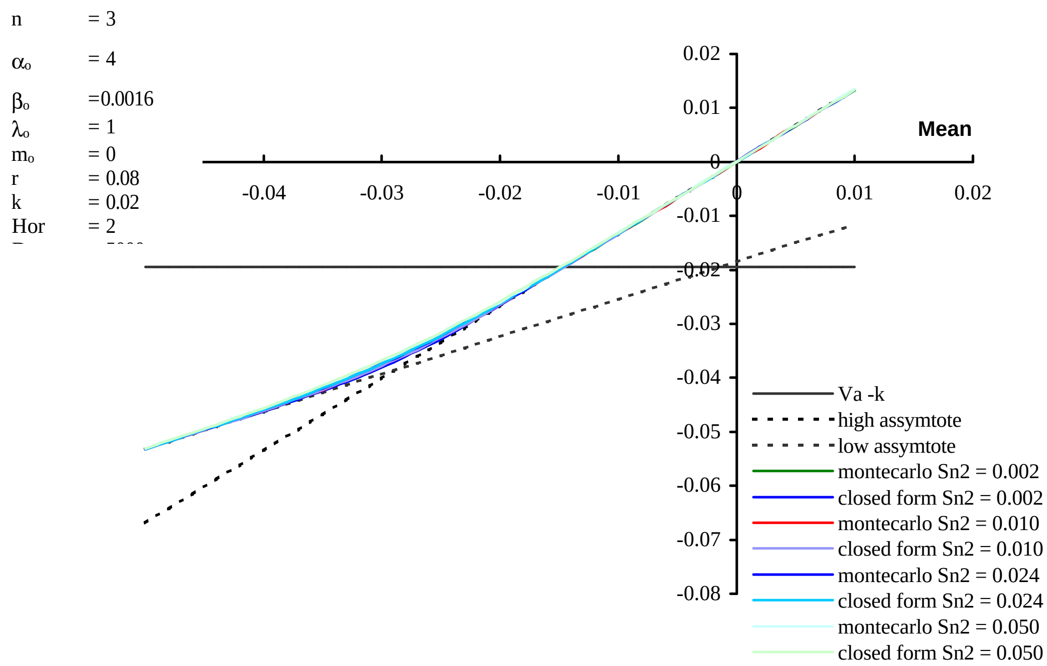

Figure 8 Montecarlo accuracy with varying numbers of draws

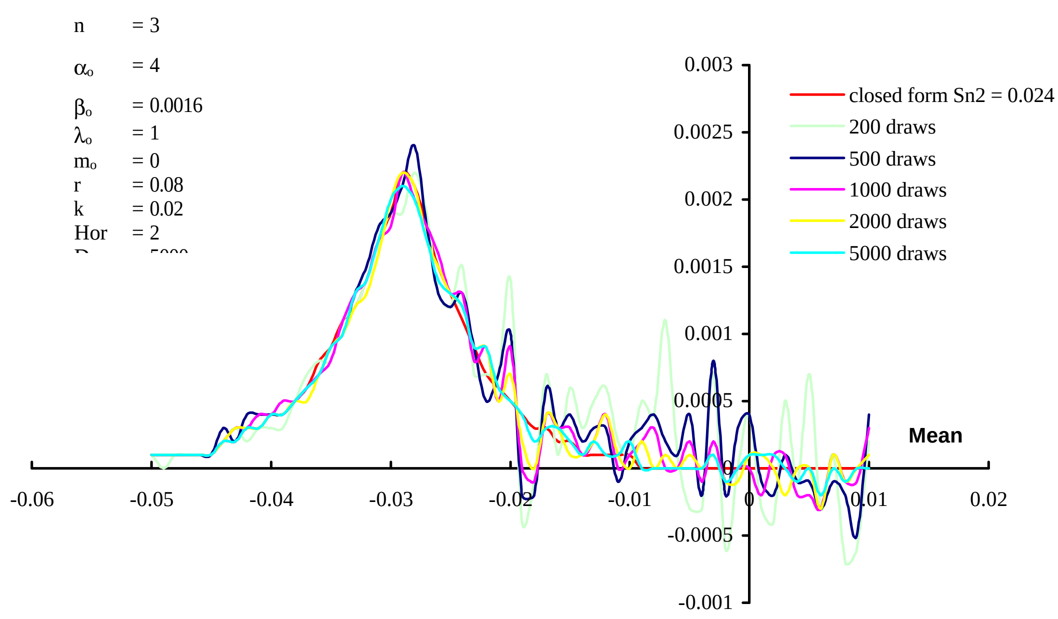

Figure 9 Value of active management for representative investor

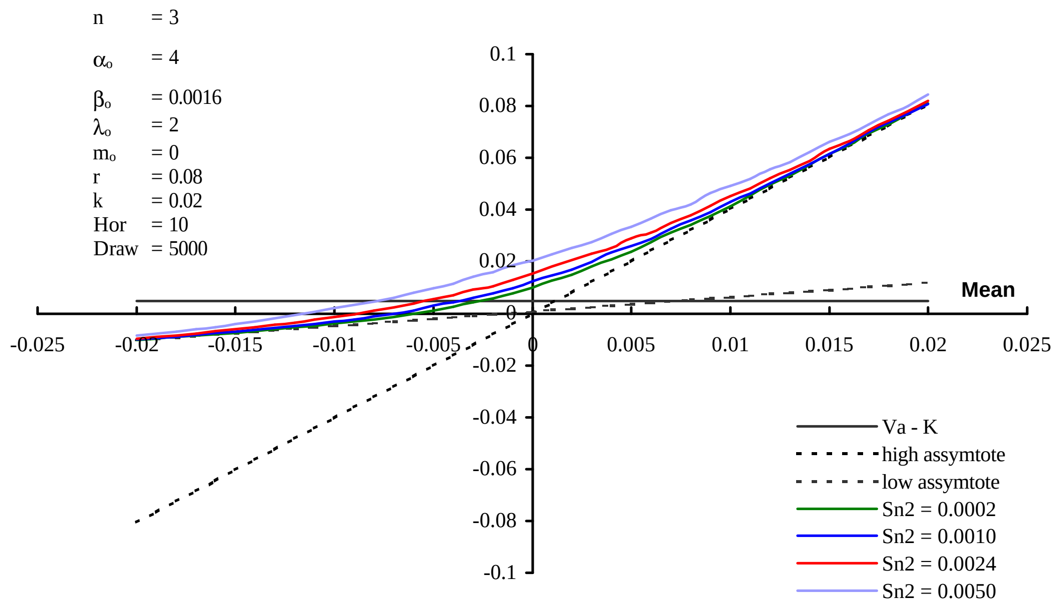

Figure 10 Variation of option value of active management

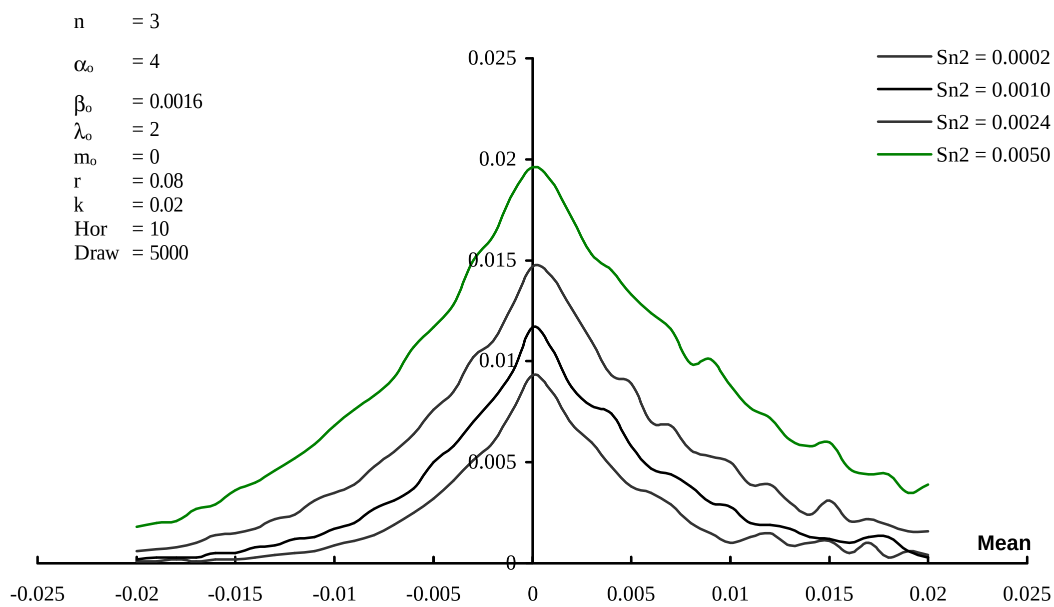

Figure 11 Variation of value with level of sample information

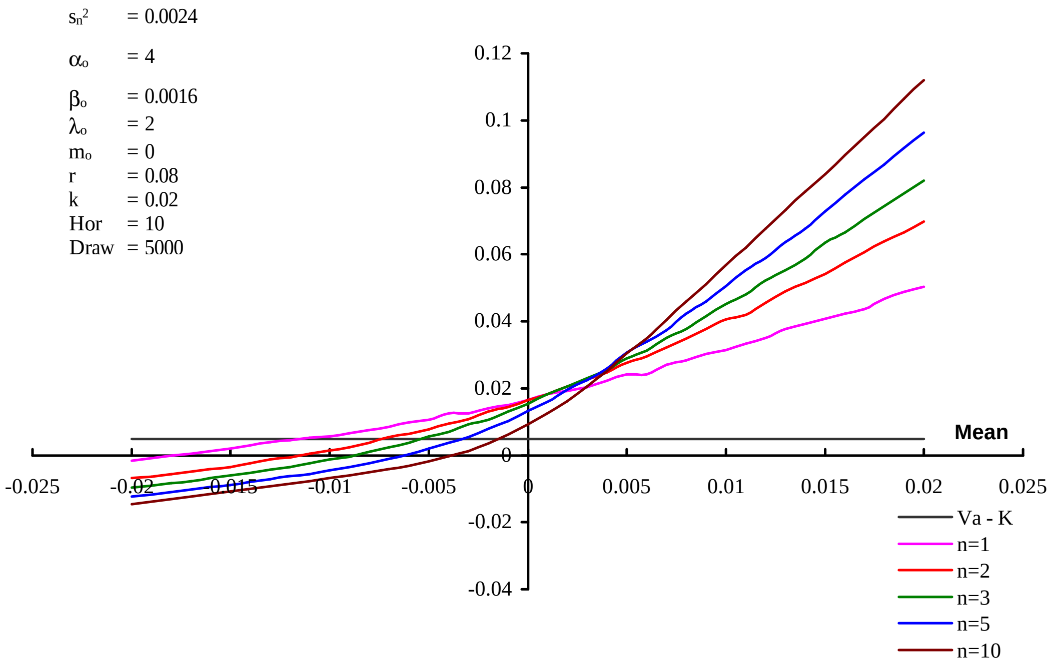

Figure 12 Option value versus level of sample information

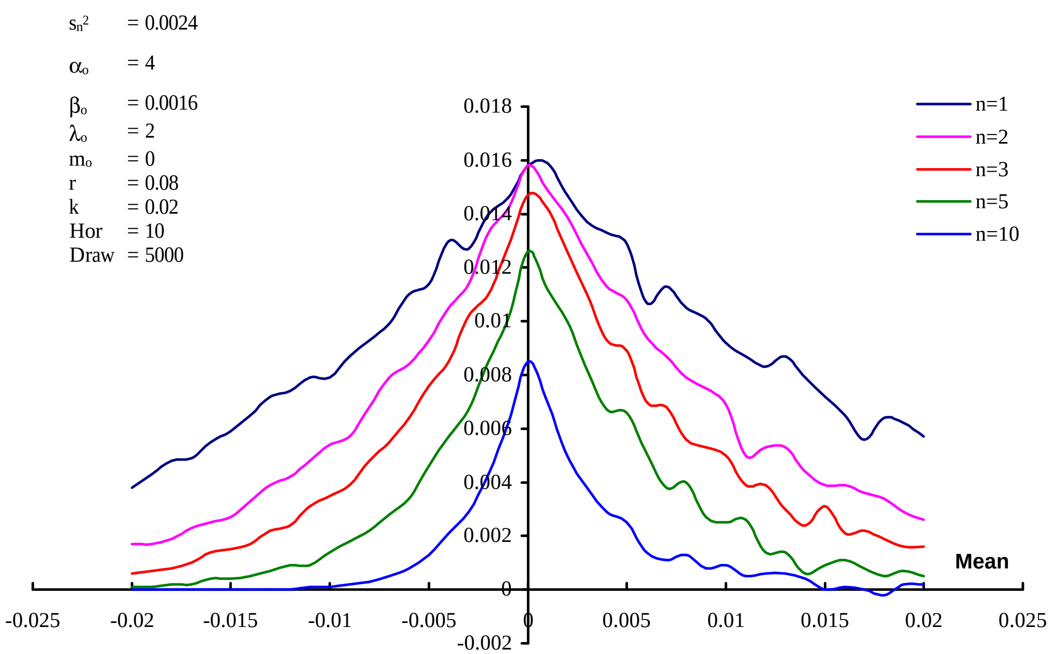

Figure 13 High uncertainty over value of μ

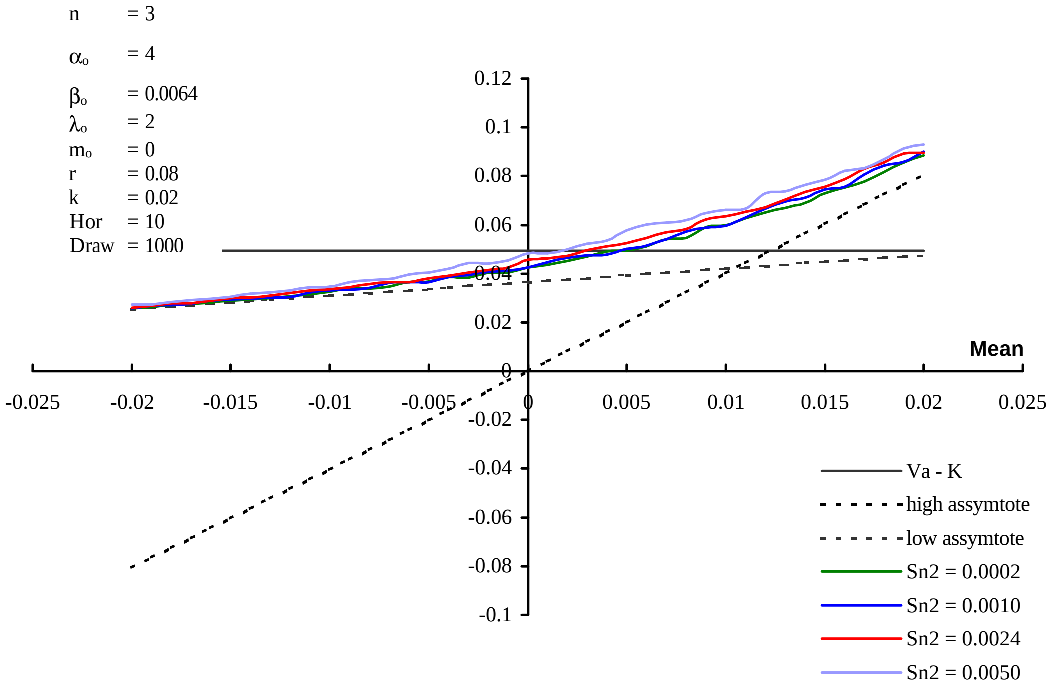

Figure 14 Low uncertainty over the value of μ

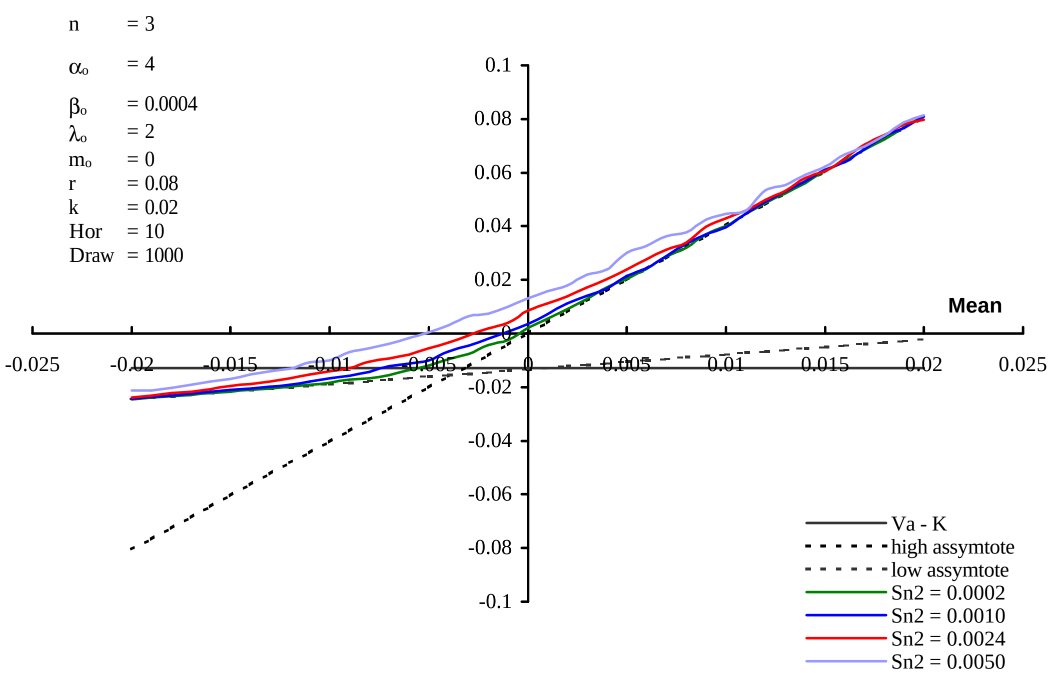

Figure 15 Strong prior information

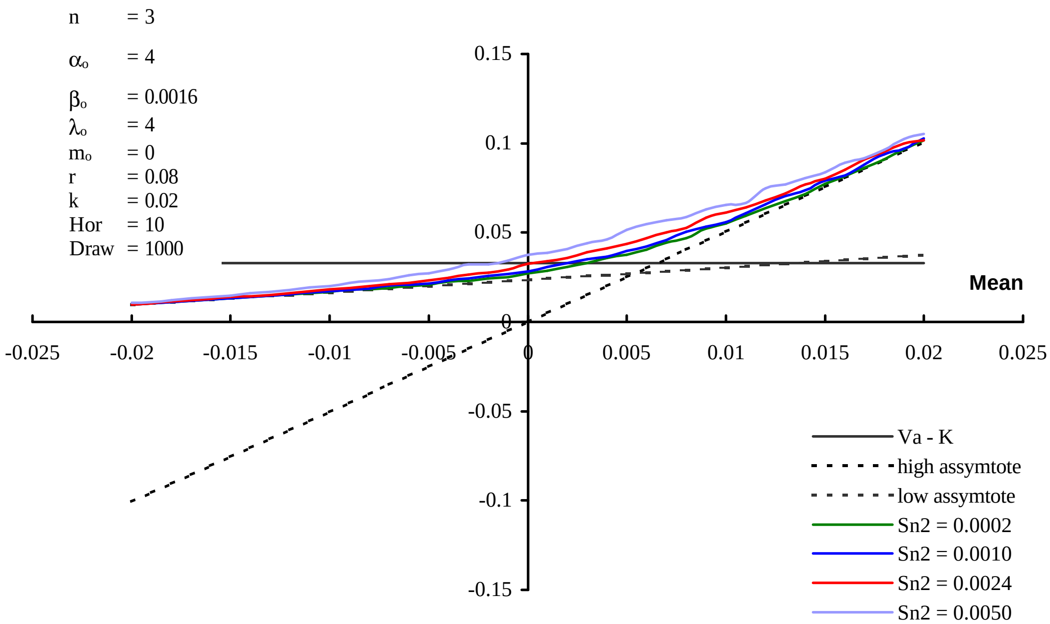

Figure 16 Weak prior information

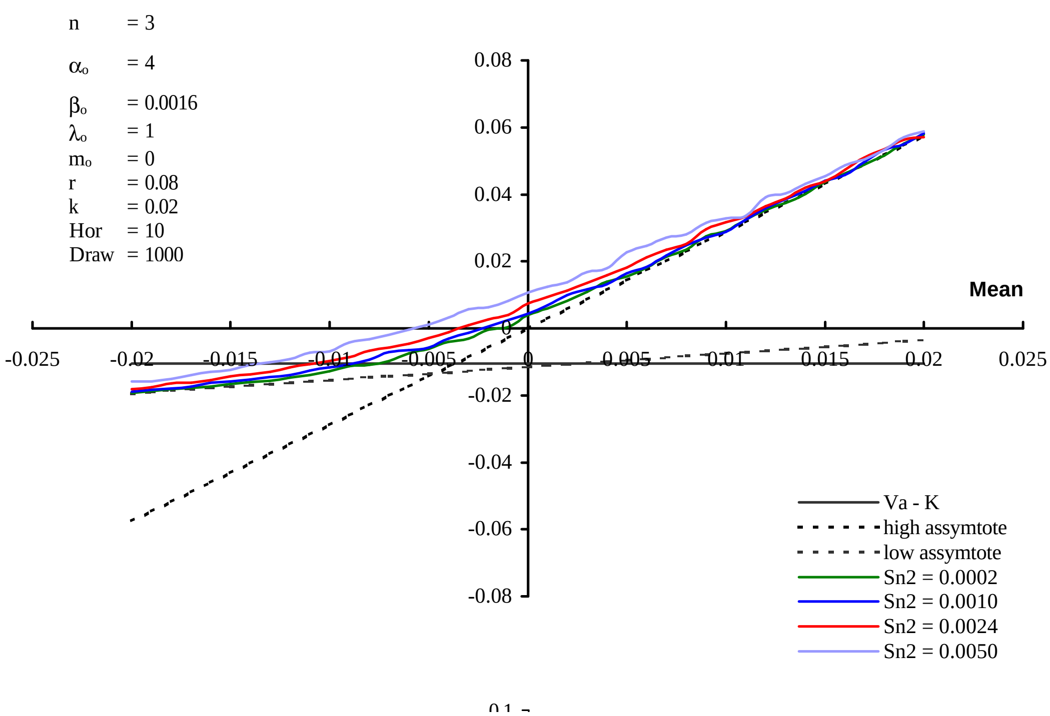

Figure 17 High cost of changing manager

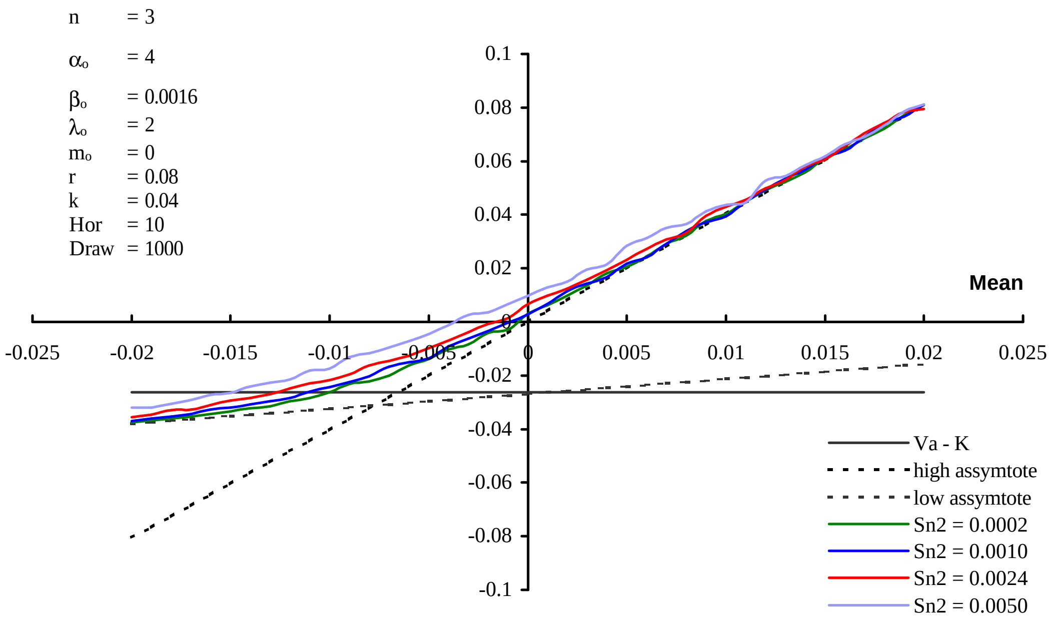

Figure 18 Low cost of changing manager

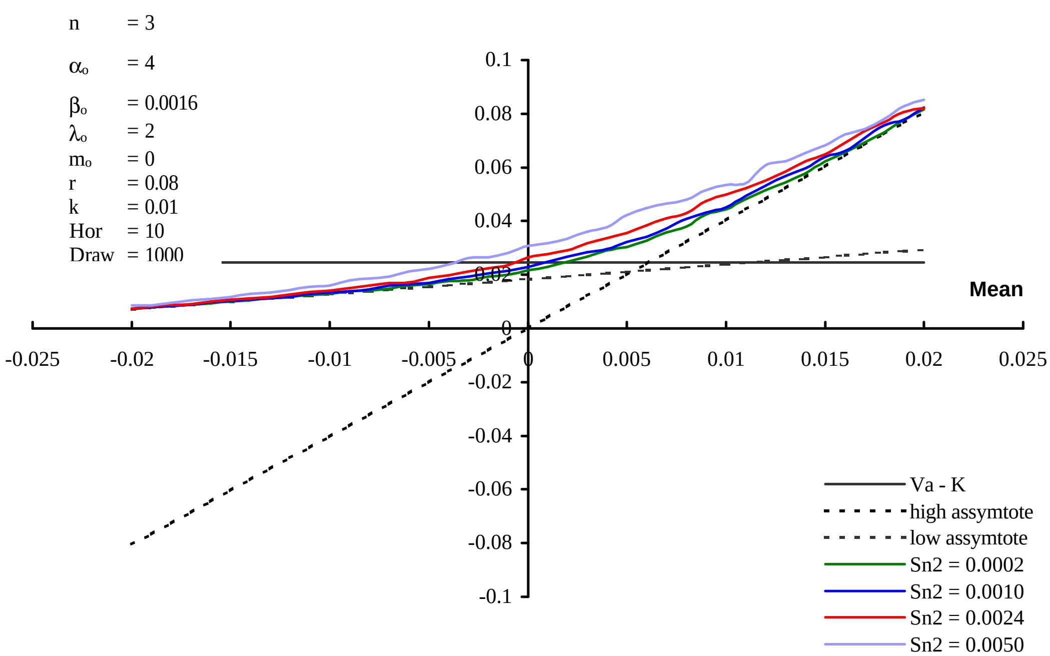

Figure 19 High risk free rate

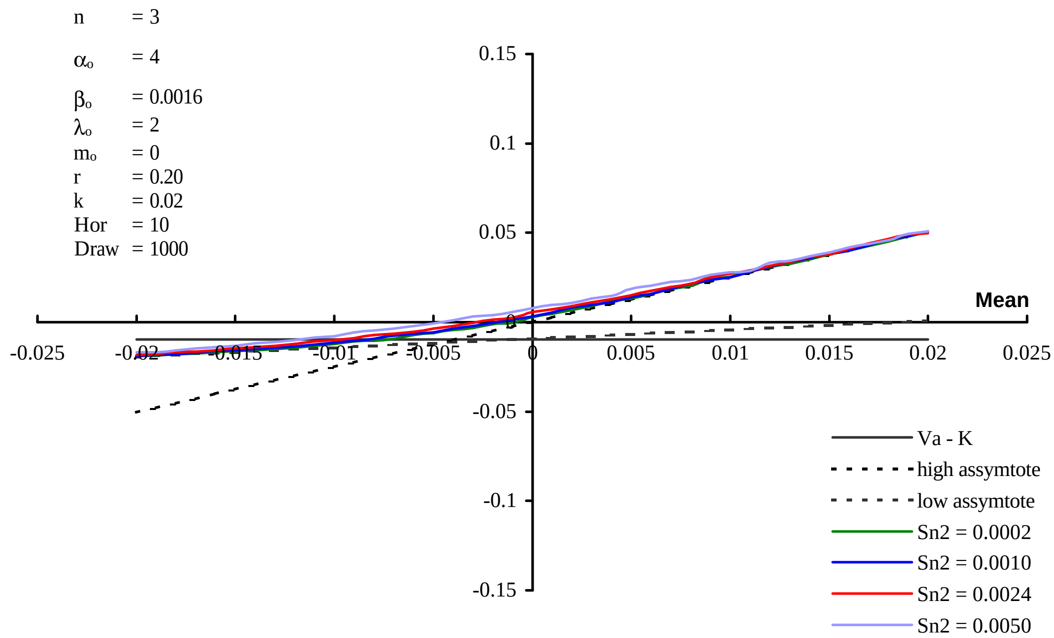

Figure 20 Low risk free rate